THE TRANSFERENCE OF HEAT BETWEEN A PIPE WALL AND A LIQUID-SOLID SUSPENSION FLOWING TURBULENTLY INSIDE THE PIPE.

THE THERMAL CONDUCTIVITY AND VISCOSITY OF A LIQUID-SOLID SUSPENSION.

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Clyde Orr, Jr.

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Approved:

Thesis Advisor

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PREFACE

Research, unlike Emerson's rhodora,* has come to require an excuse for being more specific and definable than just the beauty which is to be found in physical phenomena. Hence, when this investigation was initiated in early 1950, the elucidation of a phase of one heat transfer mechanism which had previously received scant attention was offered as the objective. It was known that diverse operations then in use required the heating or cooling of slurries, pastes or suspensions, that other applications were being considered, and that, therefore, research of the nature planned would contribute to the advancement of technology. Subsequent developments, which need not be mentioned here but which hold great promise for the future, are already making the subject one of extensive investigation. It is hoped the information presented here will find its proper use.

It would be presumptuous to suggest, in the present early stages of investigation of this subject, that all factors have been considered and taken into account. Indeed, that was not intended. It was intended only that broad, general rules which would suffice for most engineering work should be laid down. To that end a concerted effort was made in the investigation to select those materials and conditions which would bring about a wide change in the obvious and perhaps major variables such as conductivity, density, heat capacity, etc. An instrument employing

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*Rhodora! if the sages ask thee why
This charm is wasted on the earth and sky,
Tell them, dear, that if eyes were made for seeing
Then beauty is its own excuse for being.

vertical flow was selected for this work, while, at about the same time, Prof. C. F. Bonilla and associates at Columbia University were investigating one liquid-solid system in horizontal flow. Since data on their system have now been published and are included in the correlations here, considerable coverage of the field is presented.

Many individuals and organizations have contributed to this report. In particular, I wish to recognize the encouragement, advice and direction of Prof. J. M. DallaValle, who not only conceived the problem but who was ever ready to discuss its various aspects; Prof. M. J. Goglia, who generously gave of his time and experience; Prof. W. M. Newton, whose suggestions resulted in certain checks and safeguards; Dr. Paul Weber, Director of the Department of Chemical Engineering, who made it possible to obtain the equipment; and Mr. H. G. Blocker, who, with care and persistence, made most of the viscosity determinations. In addition, I wish to thank Miss Patsy Blocker for her help in making the drawings and for typing the manuscript and to acknowledge the editorial contribution of Mrs. Nancy Wastler. And last, I wish to acknowledge my gratitude to Dr. R. N. Boarts, Director of the Chemical Engineering Department of the University of Tennessee, who kindled and patiently directed my early interest in heat transfer.

SYMBOLS

a. Latin Letter Symbols

A	Cross-sectional area	$ft.^2$
a	Constant	
b	Constant	
C	Heat capacity	Btu/lb., °F.
c	Constant	
D	Inside pipe diameter	ft.
d	Particle diameter	microns
F	Fraction of solid material by volume	
f	Fanning friction factor	
g	Acceleration of gravity	$ft./sec.^2$
g_c	Conversion factor	$lb\text{-}mass, ft./lb\text{-}force, sec.^2$
H	Head of fluid	ft.
h	Average individual heat transfer coefficient	$Btu/hr., ft.^2, {}^\circ F.$
k	Thermal conductivity	$Btu/hr., ft.^2$ (°F. per ft.)
L	Length	ft.
m	Constant	
P	Pressure	$lb\text{-}force/ft.^2$
q	Quantity of heat per unit time	Btu/hr.
T	Temperature	°F.
t	Time	min.
V	Volume	$ft.^3$
v	Average linear velocity	ft./sec.
W	Fraction of solid material by weight	
w	Pipe wall thickness	ft.
x	Distance	ft.

b. Greek Letter Symbols

Δ (delta)	Finite difference	
μ (mu)	Dynamic viscosity, coefficient of	lb-mass/hr., ft.
ρ (rho)	Density	$lb\text{-}mass/ft.^3$
ϕ (phi)	Fluidity, coefficient of	hr., ft./lb-mass
σ_g (sigma)	Geometric standard deviation	

c. Latin Letter Subscripts

b	Sedimented bed
c	Constant
f	Friction
g	Geometric
i	Inlet

(Continued)

SYMBOLS (Concluded)

L	Liquid
m	Mean
o	Outlet, Zero fluidity
p	Particles, Constant pressure
s	Suspension
w	Pipe wall

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I. SUMMARY

Although the transference of heat between a pipe wall and a homogeneous liquid flowing turbulently inside the pipe is widely used and has been subjected to extensive investigation, the transference of heat between a pipe wall and a liquid-solid suspension flowing turbulently has received little attention. In the case of the homogeneous liquid, the relationship of Sieder and Tate (1936),

$$\frac{hD}{k} = 0.027 \left(\frac{Dv\rho}{\mu}\right)^{0.8} \left(\frac{C_p\mu}{k}\right)^{1/3} \left(\frac{\mu}{\mu_w}\right)^{0.14} ,$$

has been shown to describe experimental data for both heating and cooling. This relationship also describes the situation in the case of the suspensions and conditions investigated, provided that the properties of the suspension for which values are required are correctly evaluated.

No difficulty is presented in evaluating a suspension's density, ρ, and its heat capacity, C_p, since these properties are given by the weighted average of the properties of the individual components. The pipe diameter, D, and the average velocity of flow, v, are, of course, readily determinable. The thermal conductivity of the suspension may be simply evaluated from the relationship proposed by Tareef (1940),

$$k_s = k_L \left[\frac{2k_L + k_p - 2F\ (k_L - k_p)}{2K_L + k_p + F\ (k_L - k_p)} \right] ,$$

which requires only that the conductivity of the individual components at the temperature in question and their relative amounts be known. Difficulty exists only with viscosity.

The viscosity of a suspension, particularly one having a concentration of solid particles greater than a few volume per cent, is likely to vary with the rate of flow (in addition to its variation with temperature) and to depend, at any concentration of the solid particles, on particle size distribution, particle shape and even on interfacial forces of an electrical nature. Since the principle resistance to the transfer of heat between a pipe and the fluid flowing inside is the layer in streamline or laminar flow at the pipe wall, it is the viscosity manifest under this condition that is required. The relationship,

$$\mu_s = \frac{\mu_L}{(1 - \frac{F}{F_b})^{1.8}} ,$$

which was developed in the course of this investigation, satisfactorily expresses the viscosities of the suspensions investigated for the present purpose. The relationship, requiring a measure of the volume occupied by solid material in the bed which results from extended gravity sedimentation of the suspension, in a sense considers particle size distribution, particle shape and interfacial forces, and this fact is believed to be the source of the relationship's superiority over previous generalizations.

Although a suspension's viscosity in streamlined flow controls in the case of heat transfer, it is the viscosity in the turbulent region that establishes the resistance to the turbulent flow of a suspension in a pipe. In agreement with the findings of Caldwell and Babbitt (1941), this resistance was found to be predictable for the systems investigated from a Fanning friction factor versus Reynolds number plot, provided that the viscosity of the pure liquid and the density of the suspension were used in the Reynolds number.

A suspension under optimum conditions may be a somewhat better heat transfer medium than the liquid alone. However, solely as a heat transfer medium, a suspension would seem to have little to recommend it, for the disadvantages arising from the difficulties of handling more than outweigh the advantages. The principal value of this investigation lies in the fact that relationships have been set forth by which a suspension may be treated when the use of a suspension is mandatory.

II. INTRODUCTION

The transference of heat from a pipe wall to a homogeneous liquid or vice versa is of great importance in the field of power generation, to the petroleum industry, to the chemical industry in general and, in fact, to many applications involving heat, whether it be the cooling of an automobile engine, household heating or an industrial process. The subject has consequently been studied extensively. In contrast, only the study of Bonilla, et al. (1951) of heat transfer between solid surfaces and liquid-solid suspensions in which the physical properties of the components were also given and from which the mechanism of transfer might be studied has been found. This work was presented in December, 1951. However, at the time of initiation of this study (April, 1950) a short investigation, made by students of Prof. H. C. Lewis, indicated that the over-all coefficient of heat transfer between a steam-heated pipe and an oil-sand suspension was somewhat greater than that for the oil alone.

The subject of the heat transfer properties of suspensions appeared to be worthy of further investigation for several reasons. First, the extremely high thermal conductivity of solids in comparison with the conductivity of some liquids suggested that a combination such as could be brought about in a suspension might enhance the heat transfer characteristics to a significant degree. Second, the variation in many properties that could be brought about by the use of a suspension afforded a previously unavailed opportunity to study the heat transfer mechanism. Third, it was desired to determine whether rapidly moving and relatively dense particles could partially destroy the principle resistance to heat transfer in the situation to be investigated, namely, the film at the pipe surface. Fourth, the existing usage of suspensions needed to be placed on an established basis. And finally, future usage, which it was felt might be extensive, would require information of the nature contemplated. For these reasons, a study of the transference of heat from a pipe wall to a suspension was undertaken.

The great body of information dealing with the flow and heat transfer properties of pure liquids forms an ideal basis from which to examine suspensions since pure liquids are a limiting case of suspensions. It is now generally agreed that, when a liquid flows in a pipe with a mean velocity exceeding a certain value, three zones or types of flow--film, buffer and turbulent--exist simultaneously. Film or streamline flow prevails near the wall, turbulent flow prevails in the core, and between these zones the buffer or intermediate layer exists. Heat leaving the pipe wall must be conducted through the film and into the buffer layer. In the latter, heat is both conducted and transferred by mechanical mixing into the turbulent core comprising the major portion of the liquid.

When these characteristics are considered, a grouping of logical factors in relation to the average individual coefficient of heat transfer, h, may be obtained. Since heat must be conducted through the liquid film, the conductivity of the liquid, k, will be a factor. The film thickness will depend upon the velocity of flow, v; the liquid's viscosity, μ ; and upon the pipe diameter, D. Hence, these factors must be considered. Furthermore, because as a given amount of heat is transferred, the heat capacity, C_p (nearly incompressible fluids are being considered), affects the bulk temperature of the stream, this parameter must also be considered. As shown by McAdams (1942), a treatment of these factors according to the principles of dimensional analysis results in the relationship,

$$\frac{hD}{k} = a\left(\frac{Dv\rho}{\mu}\right)^b \left(\frac{C_p\mu}{k}\right)^c , \qquad (1)$$

where a, b and c are dimensionless constants whose values can only be experimentally determined.

Under conditions of turbulent flow and for viscosities not exceeding twice that of water, F. W. Dittus and L. M. K. Boelter, as quoted by McAdams (1942), have suggested that equation 1 be used with a having a value of 0.023, b having a value of 0.8 and c having a value of 0.4 if the liquid is being heated and 0.3 if the liquid is being cooled. Regardless of whether the liquid is being heated or cooled, the liquid's properties are to be evaluated at its bulk mean temperature. Since viscosity is the only property that varies significantly with temperature, other suggestions, especially applicable to liquids of higher viscosities, have been offered. Colburn (1933) shows that the use of equation 1 with a and b having values of 0.023 and 0.8, respectively, with c having a value of 1/3 and with all properties evaluated at the bulk liquid temperature except viscosity, which is evaluated at the film temperature, is more logical and applicable. Evaluating the film temperature presents difficulties. (It is often taken as the average of the bulk liquid and pipe wall temperatures, however.) Therefore, Sieder and Tate (1936) have shown that the equation suggested by Colburn is satisfactorily approximated and its use is made more convenient if the ratio, raised to the 0.14 power, of the liquid viscosity at the bulk mean liquid temperature to the liquid viscosity at the temperature of the inner surface of the pipe wall is incorporated, if all other terms are evaluated at the bulk temperature and if the constant a is given the value of 0.027. The resulting equation of Sieder and Tate is

$$\frac{hD}{k} = 0.027\left(\frac{Dv\rho}{\mu}\right)^{0.8} \left(\frac{C_p\mu}{k}\right)^{1/3} \left(\frac{\mu}{\mu_w}\right)^{0.14} . \qquad (2)$$

As with the Colburn equation, the use of equation 2 is not recommended for Reynolds numbers of less than 10,000.

Because equation 2 may be accepted for use with all liquids within the limits outlined above, and because it can be conveniently used, it will be employed extensively in the discussions that follow. Indeed, it will be shown to describe the situation in the case of liquid-solid suspensions over a quite considerable range. A large part of this report is concerned with showing how to evaluate the parameters comprising equation 2.

The investigation as it evolved came to have three rather distinct aspects: the heat transfer investigation proper, an investigation of the thermal conductivity of suspensions and an investigation of the viscosity characteristics of suspensions. The organization of the material to follow will accordingly be found divided along these lines until the final sections correlating and discussing all results are reached.

III. INVESTIGATIONS AND RESULTS

A. Heat Transfer

1. Apparatus

While not following it in detail, the design of the heat transfer apparatus was strongly influenced by that of the apparatus used by Martinelli et al. (1942). The over-all apparatus is shown in Figure 1 by a photograph and in Figure 2 by a schematic diagram. It consisted essentially of (1) a heat transfer section, (2) a flow-straightening device, (3) a suspension mixing chamber, (4) a suspension cooler, (5) a suspension storage and mixing tank, (6) a suspension circulating pump, (7) a suspension rotameter, (8) a pressure drop measuring system, (9) a steam separator, (10) a steam calorimeter, (11) steam pressure reducing and regulating valves, (12) steam traps, (13) a condensate-measuring system, (14) a potentiometer with its auxiliary equipment and (15) the necessary piping, valves and pressure gauges. The construction and function of each of these principal components will be discussed in detail below.

As will be noticed, the heat transfer section was mounted so that vertical, upward flow was required. The heat transfer section itself consisted of three elements—a nominal 3/8-inch standard I.P.S. copper pipe, a surrounding steam jacket of nominal 4-inch iron pipe and fittings, and a nominal 2-inch iron pipe and fittings separating the copper pipe and the outside jacket except for a small passage at the upper end of the section. The purpose of this inner wall was to separate the steam condensed on the copper pipe from that condensed on the outside wall. Effectively, this made the exchange section adiabatic. Sixteen copper-constantan

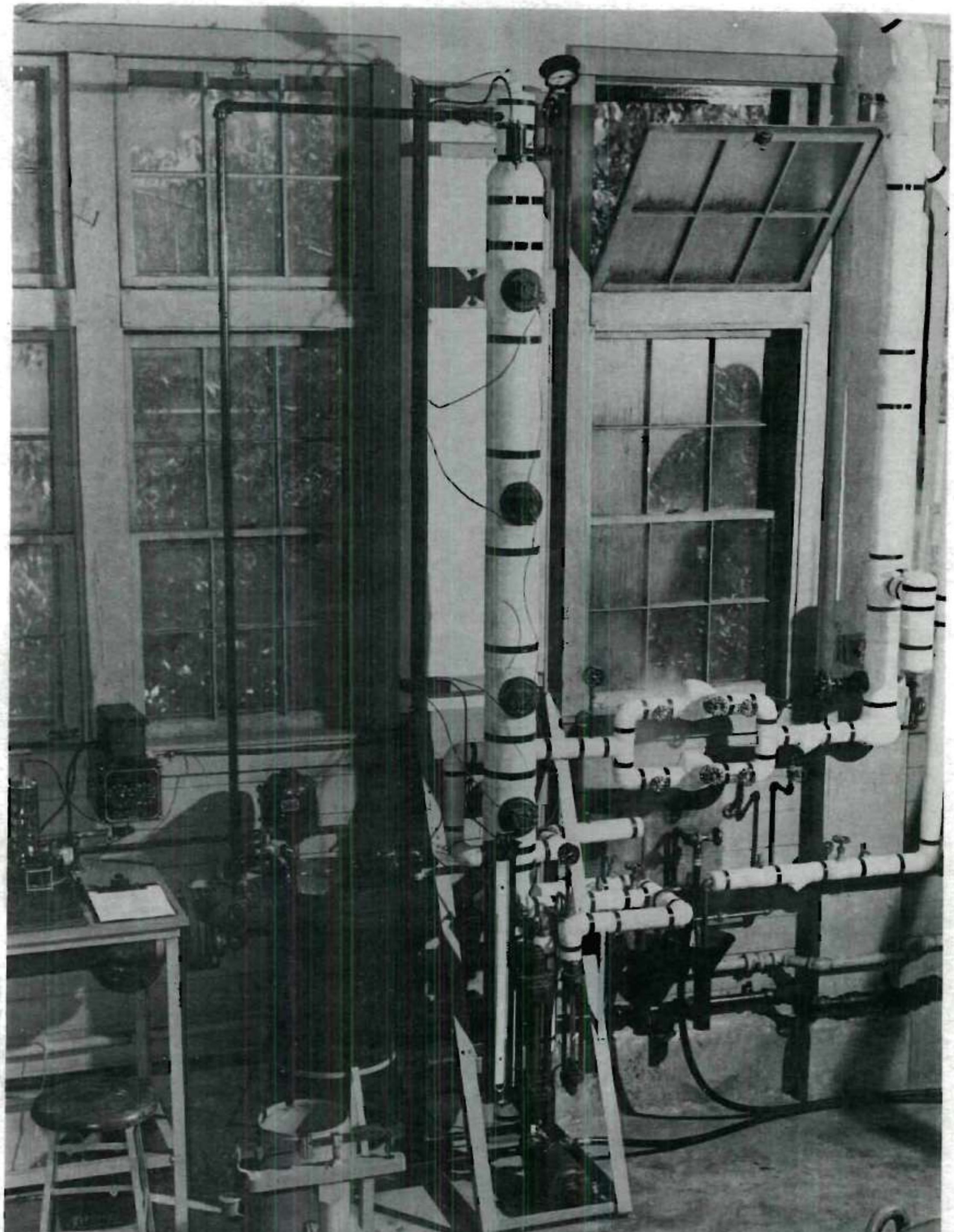


Figure 1. Heat Transfer Apparatus.

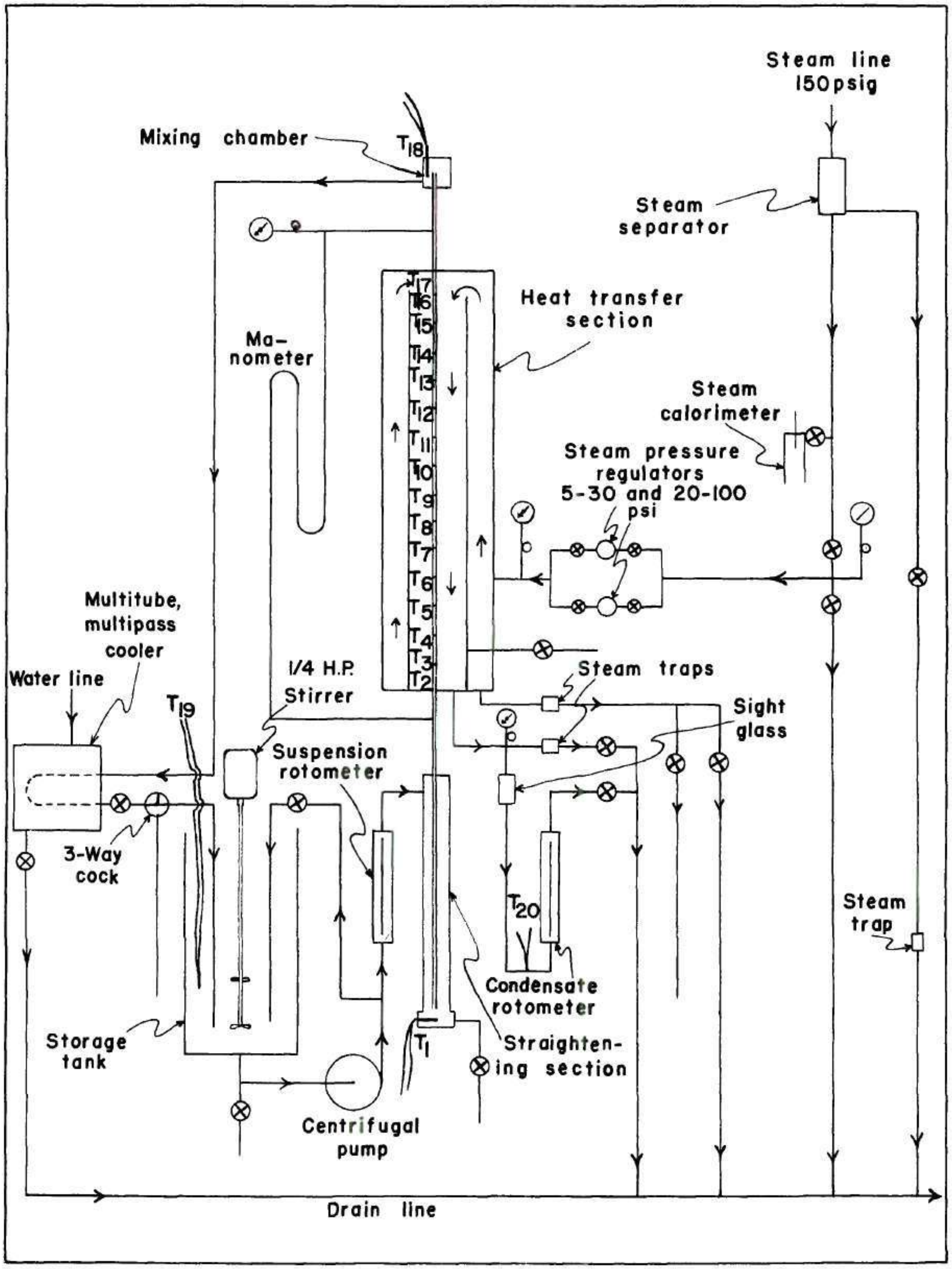


Figure 2. Schematic Diagram of Entire Heat Transfer Apparatus.

thermocouples of No. 24 wire were buried in the copper pipe wall, and their leads were brought out through both the separating and outer walls of the exchange section. Twelve of the thermocouples were located in grooves milled circumferentially around the pipe, while four thermocouples were located in grooves milled longitudinally in the pipe. In both cases the grooves were milled 3/32 of an inch deep and 1/8 of an inch wide; the circumferential grooves extended half around the pipe, while the longitudinal grooves were more than 2 inches long. The installation of each thermocouple in a groove followed a definite pattern. The thermocouple, insulated to the bead with a glass cloth shield, was placed in the groove with the bead near one end and the leads extending out the other; the bead was carefully peened into the copper pipe; a strip of copper of the width of the groove was pressed into the unfilled portion of the groove; the strip was soldered into place with pure tin (in order not to melt at the highest temperatures available); and the excess tin and copper were filed away to the level of the original pipe surface. While the thermocouple leads were located in the pipe wall for at least an inch in every case, at least another four inches remained in the steam cavity between the copper pipe and the separating wall and several more inches of leads were in the steam cavity between this separating wall and the outer jacket.

Figure 3 shows the copper pipe and one thermocouple installation located within the steam jacket; Figures 4 and 5 show progressive closing of the port through which thermocouple leads were withdrawn. The construction of the port is shown in detail in Figure 6. Four such ports

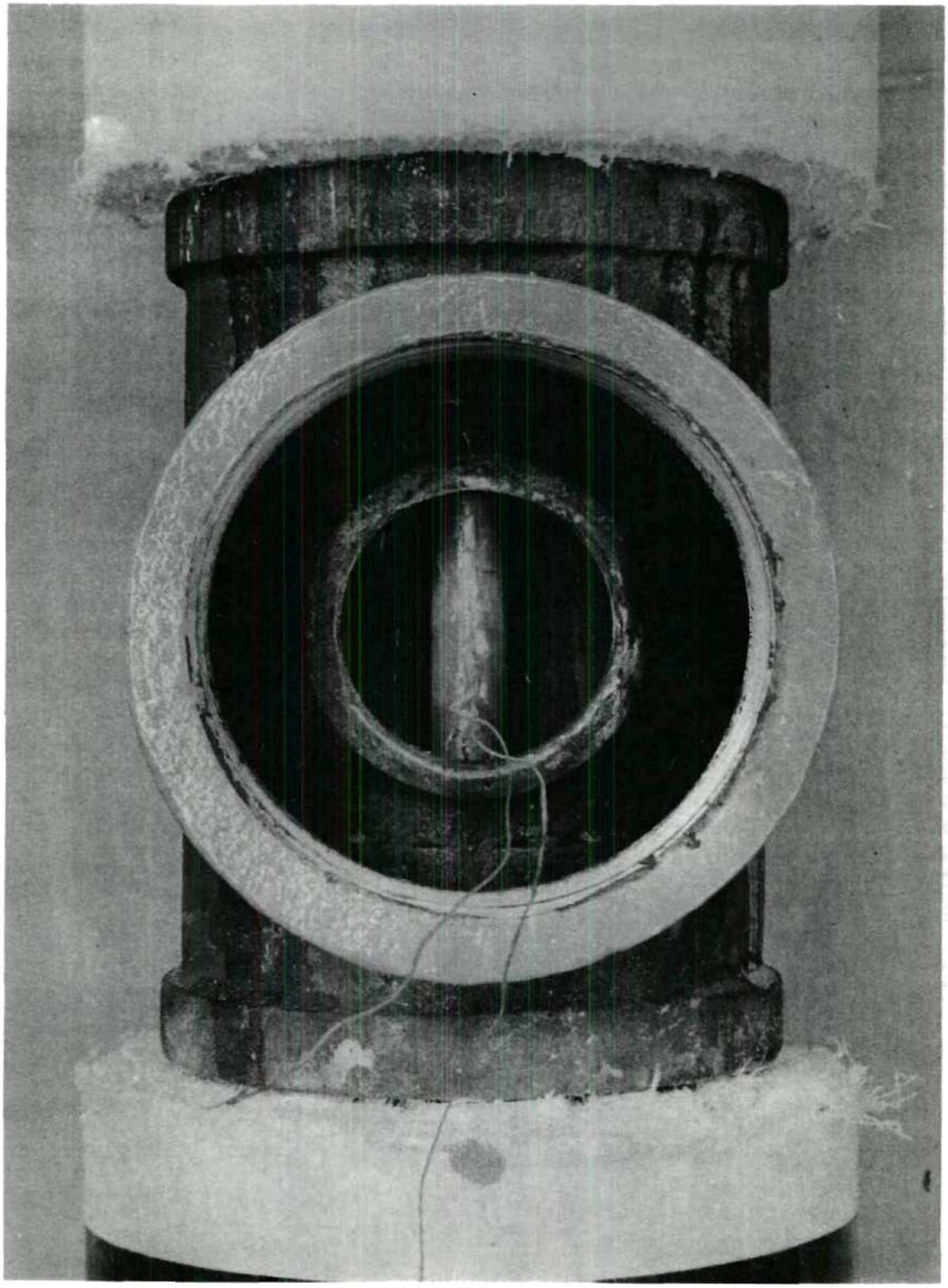


Figure 3. Thermocouple Port Completely Open Showing One Thermocouple Installed in Pipe Wall.

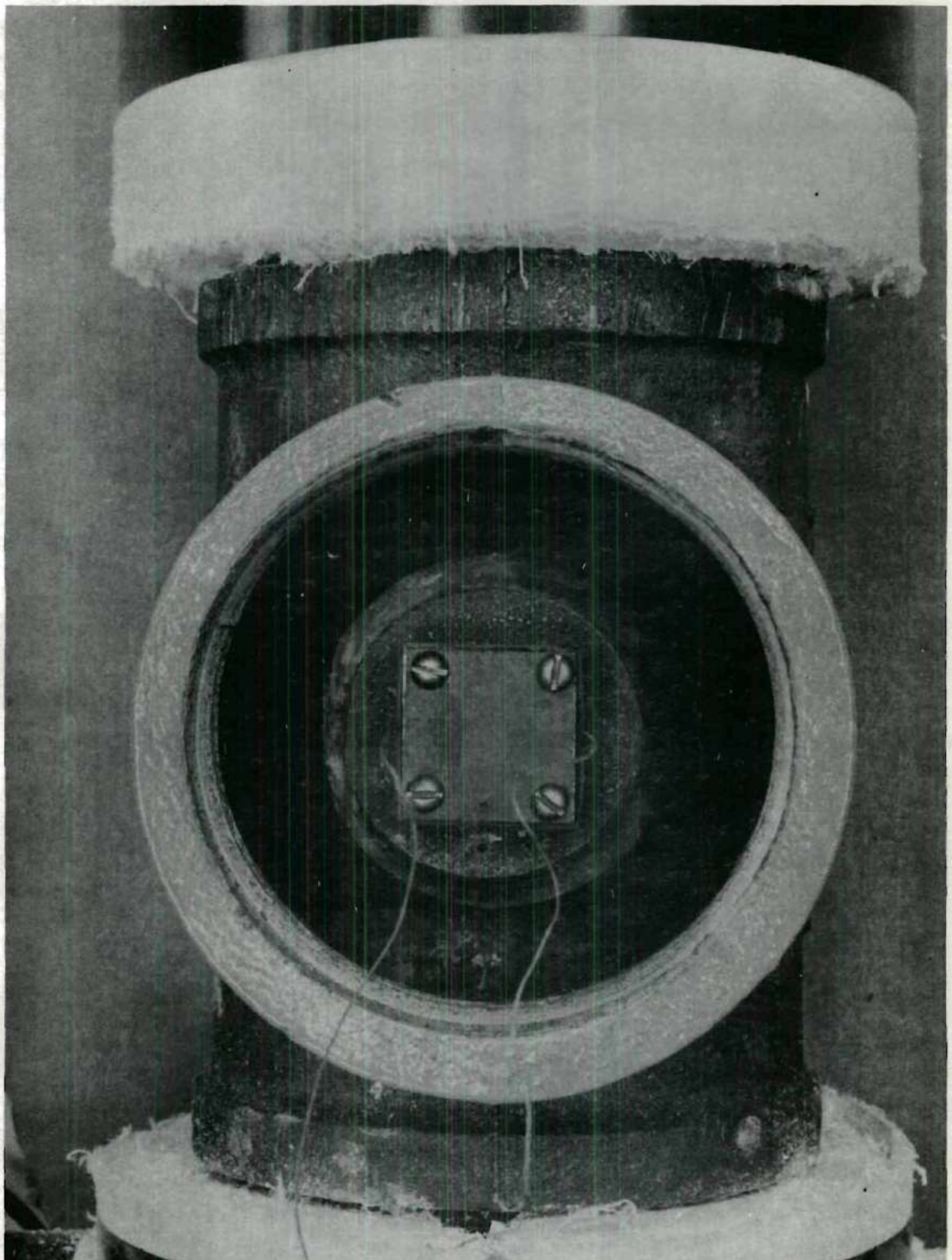


Figure 4. Thermocouple Port Partially Closed.

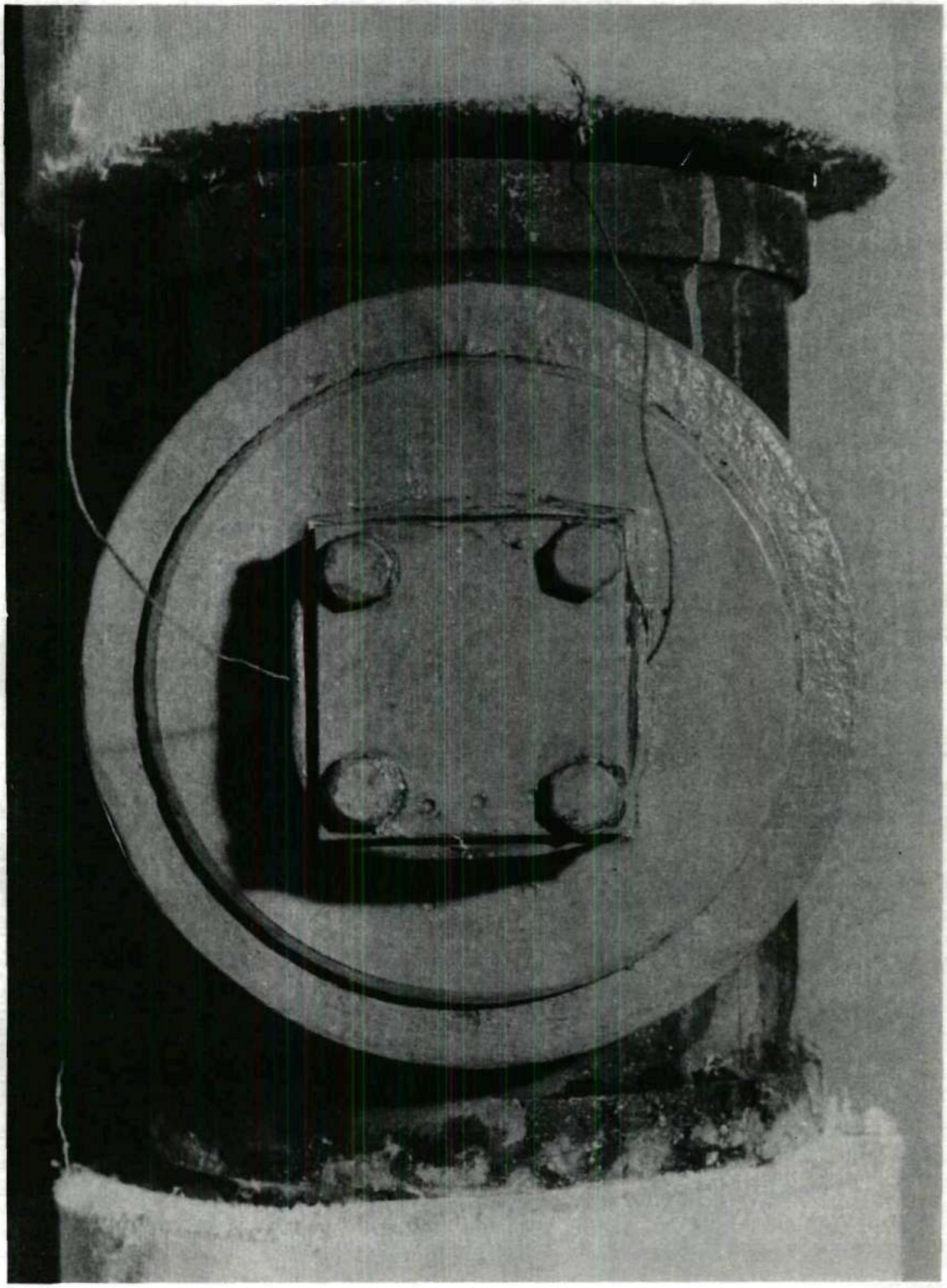


Figure 5. Thermocouple Port Completely Closed. (In the completed apparatus, leads for four thermocouples were brought out of one port.)

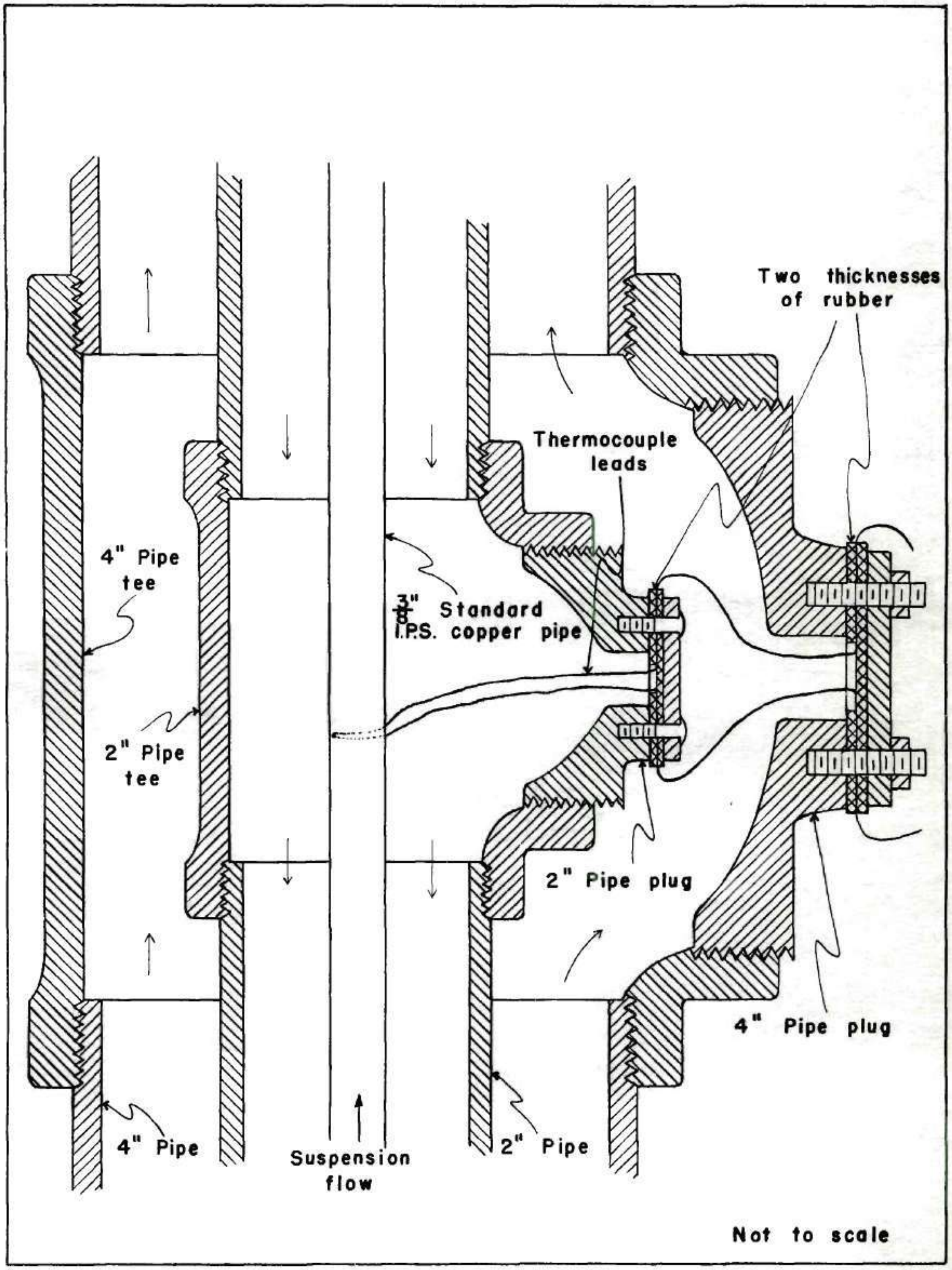


Figure 6. Details of Construction of One of the Four Ports in the Heat Transfer Apparatus Through Which Thermocouple Leads Were Withdrawn.

were provided along the heat exchange section; leads for four thermocouples were withdrawn through each port in the completed apparatus. The construction of the top and bottom ends of the heat exchange section are shown in Figures 7 and 8, respectively. The copper pipe was exposed to condensing steam for a total length of 7.70 feet. The spacing of the thermocouples over this distance was not quite uniform; the spacing of the thermocouples may be deduced from the figure showing the temperature distribution that was obtained in a representative experiment (see page 43). The entire exchange section was insulated with standard thickness, 85 per cent magnesia pipe insulation.

The copper pipe through which heat was transferred in the exchange section extended beyond each end of the exchange section. At the lower end, this extension amounted to 34-1/2 inches and served as a straightening section for the flowing liquid or suspension of about seventy pipe diameters. The details of construction of this straightening section are shown in Figure 9; the arrangement of this section, along with the location of other components of the lower portions of the apparatus, is shown in Figure 10. It will be noted that the fluid leaving the pump flowed upward through a rotameter, downward through the annular space of the straightening device and finally flowed upward again through the copper tube into the heat exchange section. Since the fluid flowing downward formed an insulating sheath about the copper pipe and since the straightening section was insulated in addition, the temperature of the fluid entering the heat transfer section was measured with a thermocouple installed in the lower part of the straightening section.

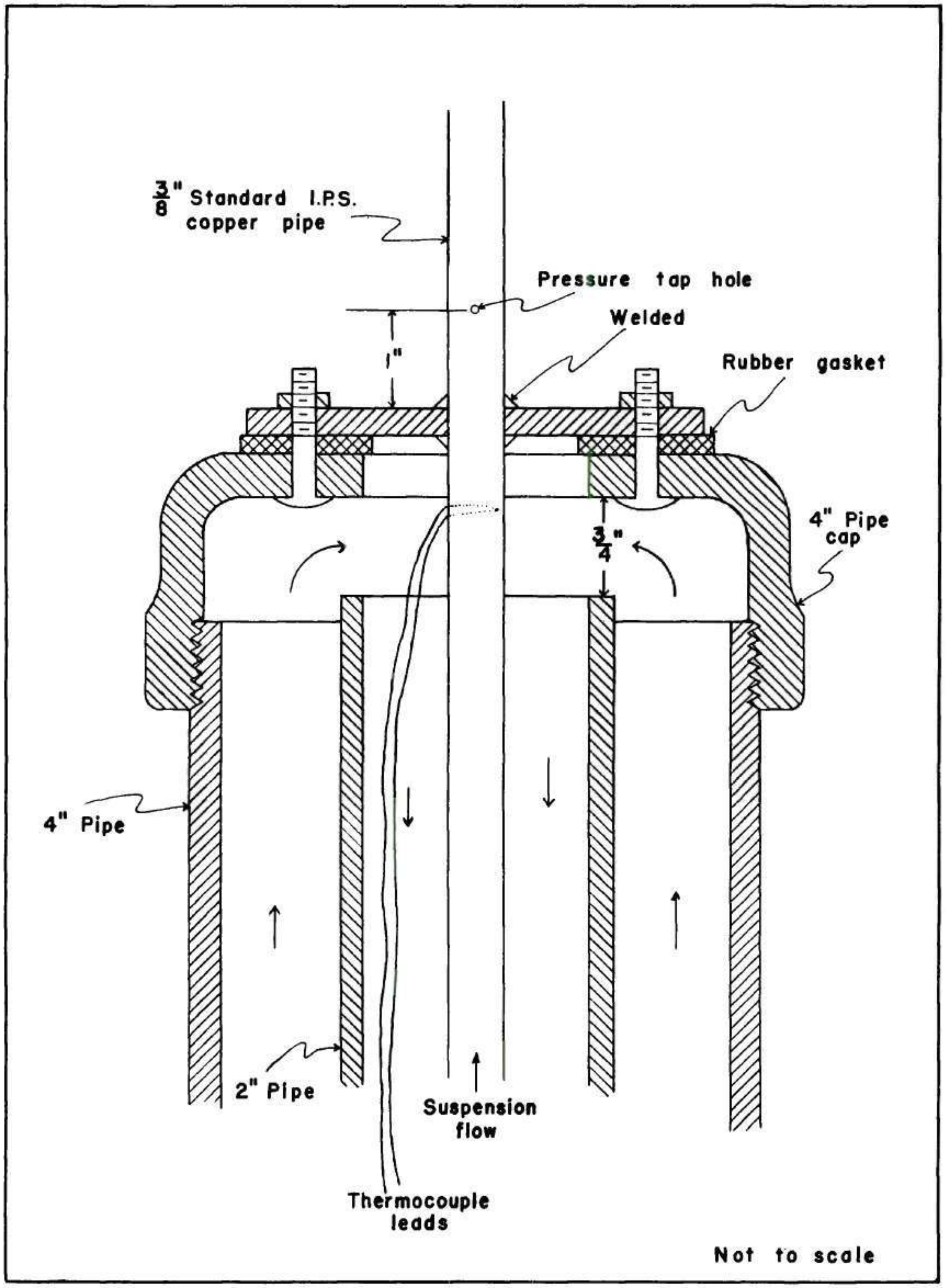


Figure 7. Details of Construction of Upper End of Heat Transfer Apparatus.

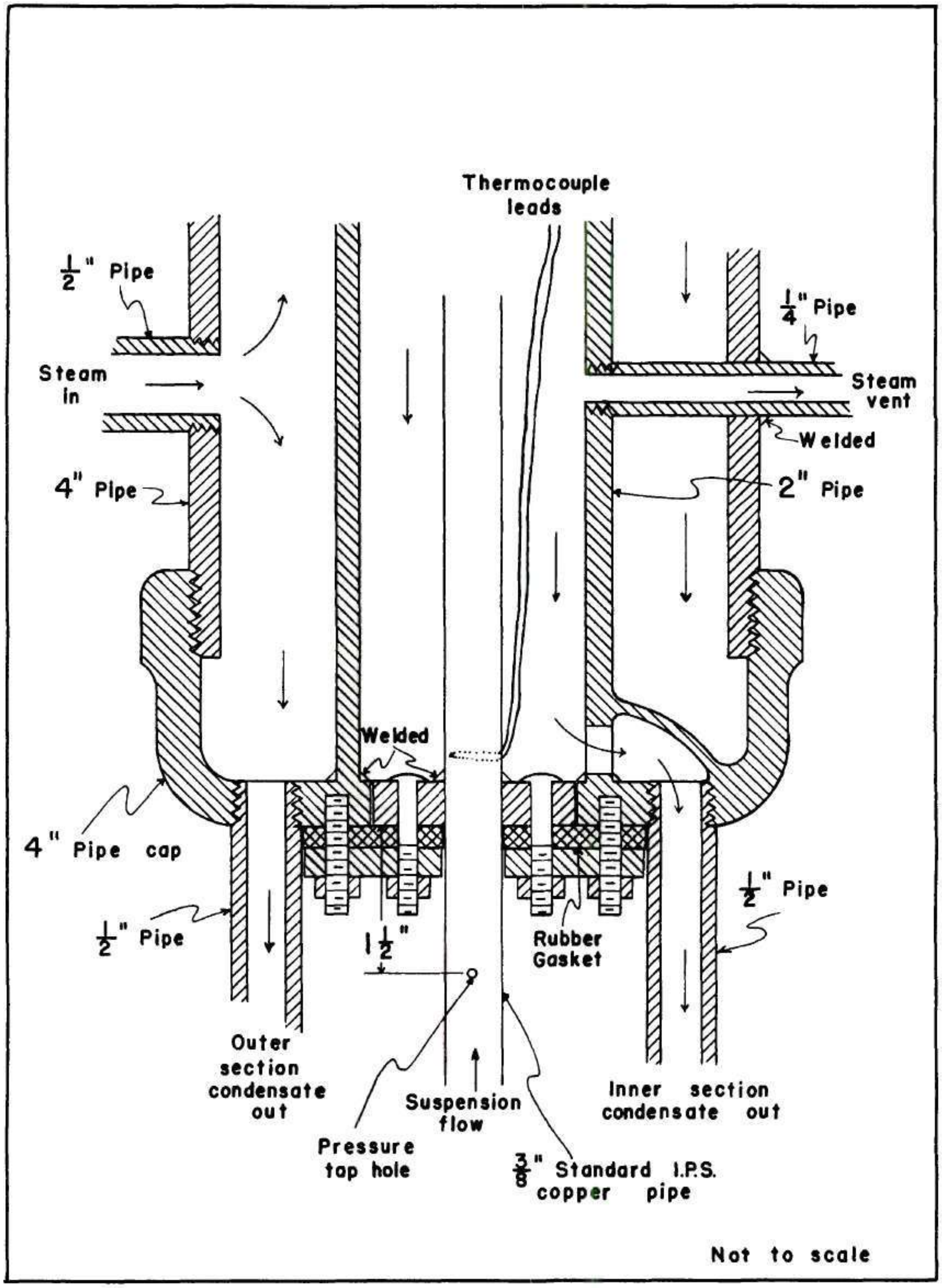


Figure 8. Details of Construction of Lower End of Heat Transfer Apparatus.

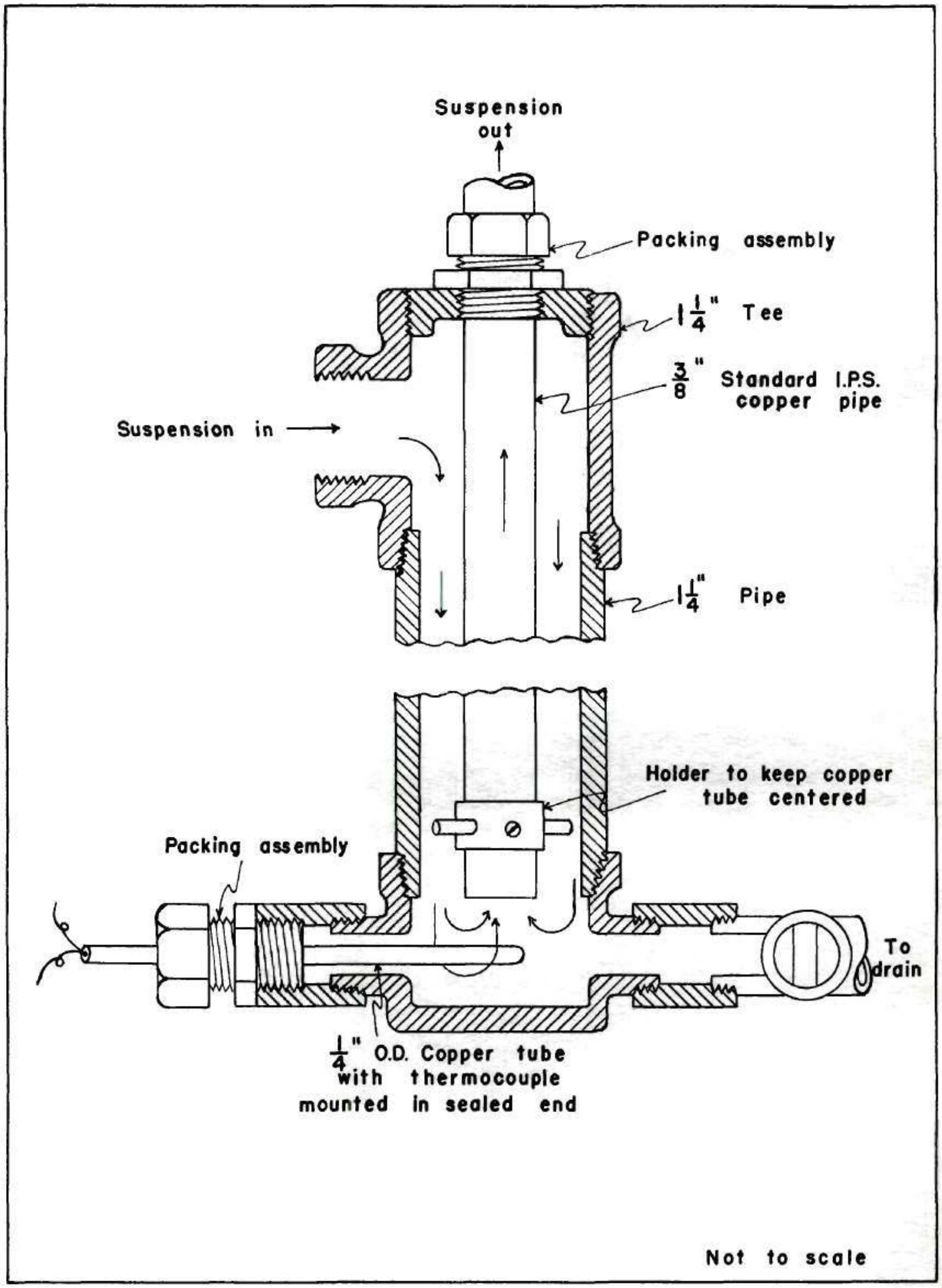


Figure 9. Details of Construction of Flow-Straightening Section.

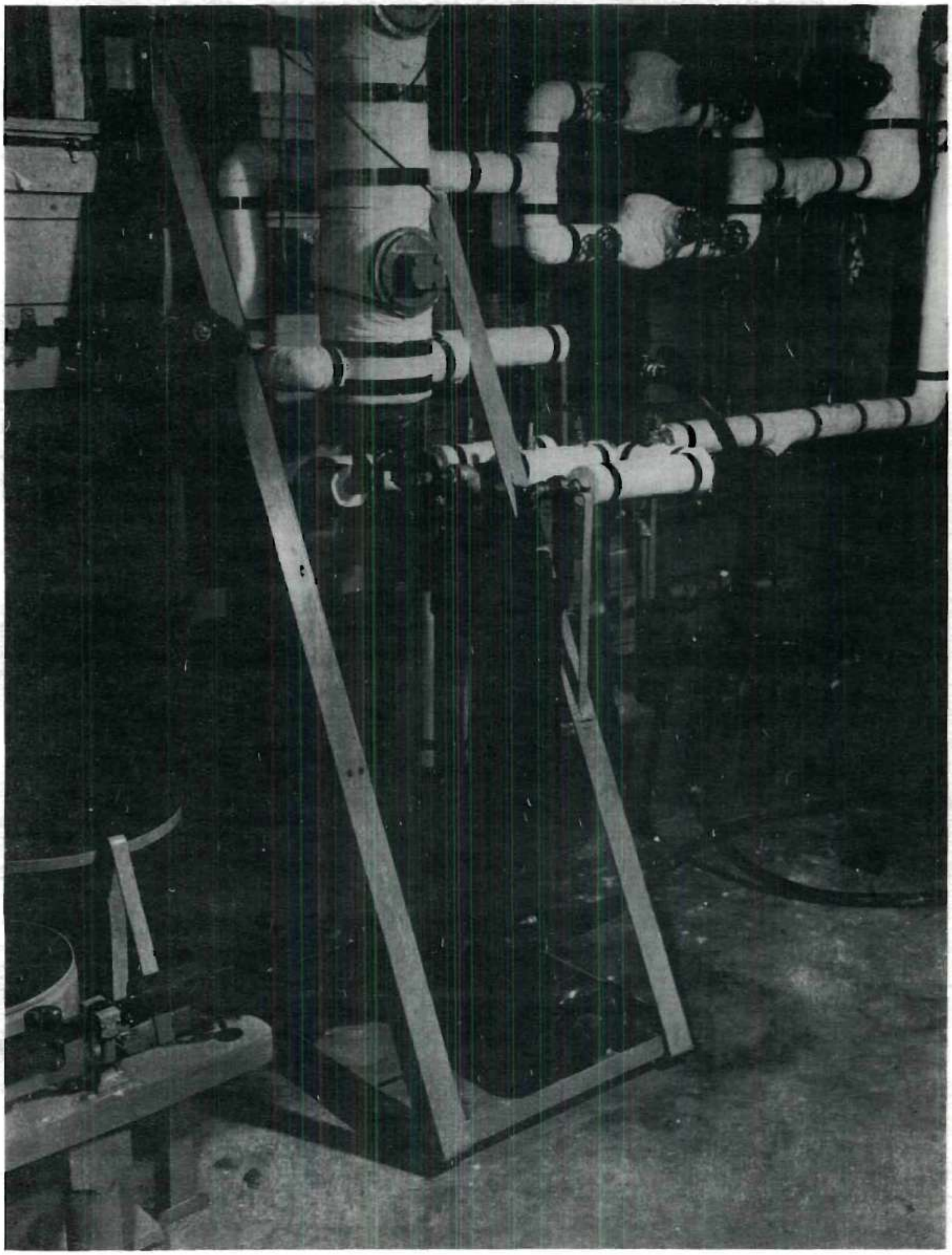


Figure 10. Lower Portion of Heat Transfer Apparatus Before the Complete Addition of Insulation.

Installation at any point nearer the heat transfer section would have disturbed the flow pattern.

The vertical copper pipe extended only 6-3/4 inches beyond the upper end of the heat exchange section. It terminated inside a chamber designed to mix the fluid so that the fluid's bulk leaving temperature might be obtained. The location of this equipment is shown during construction in Figure 11; a detailed drawing of the mixing chamber is shown in Figure 12. As may be seen, the fluid issuing from the copper pipe impinged upon the upper head of the mixing chamber, flowed spirally downward and passed out horizontally near the base of the chamber. In so doing it passed a thermocouple.

Continuing around the system, the fluid next passed through a five-tube, four-pass heat exchanger having 3/4-inch, No. 18 B.W.G. tubes and a total of 19.6 square feet of bare copper heat transfer surface. The exchanger was manufactured by the Bell and Gossett Company, Morton Grove, Illinois. The suspension passed inside the tubes, and tap water, employed as the cooling medium, passed outside the tubes.

Upon leaving the heat exchanger, where its temperature was considerably reduced, the fluid flowed to a stainless steel mixing and storage tank. In passage, it flowed through a three-way cock by which it might be diverted from time to time into a smaller weighing tank. The mixing tank was provided with a 1/4-HP, 1,725-rpm electric motor driving two propellers four inches in diameter on one shaft. The mixer was manufactured by the Alsop Engineering Corporation, Milldale, Connecticut. A variable transformer was provided so that the violence of agitation might

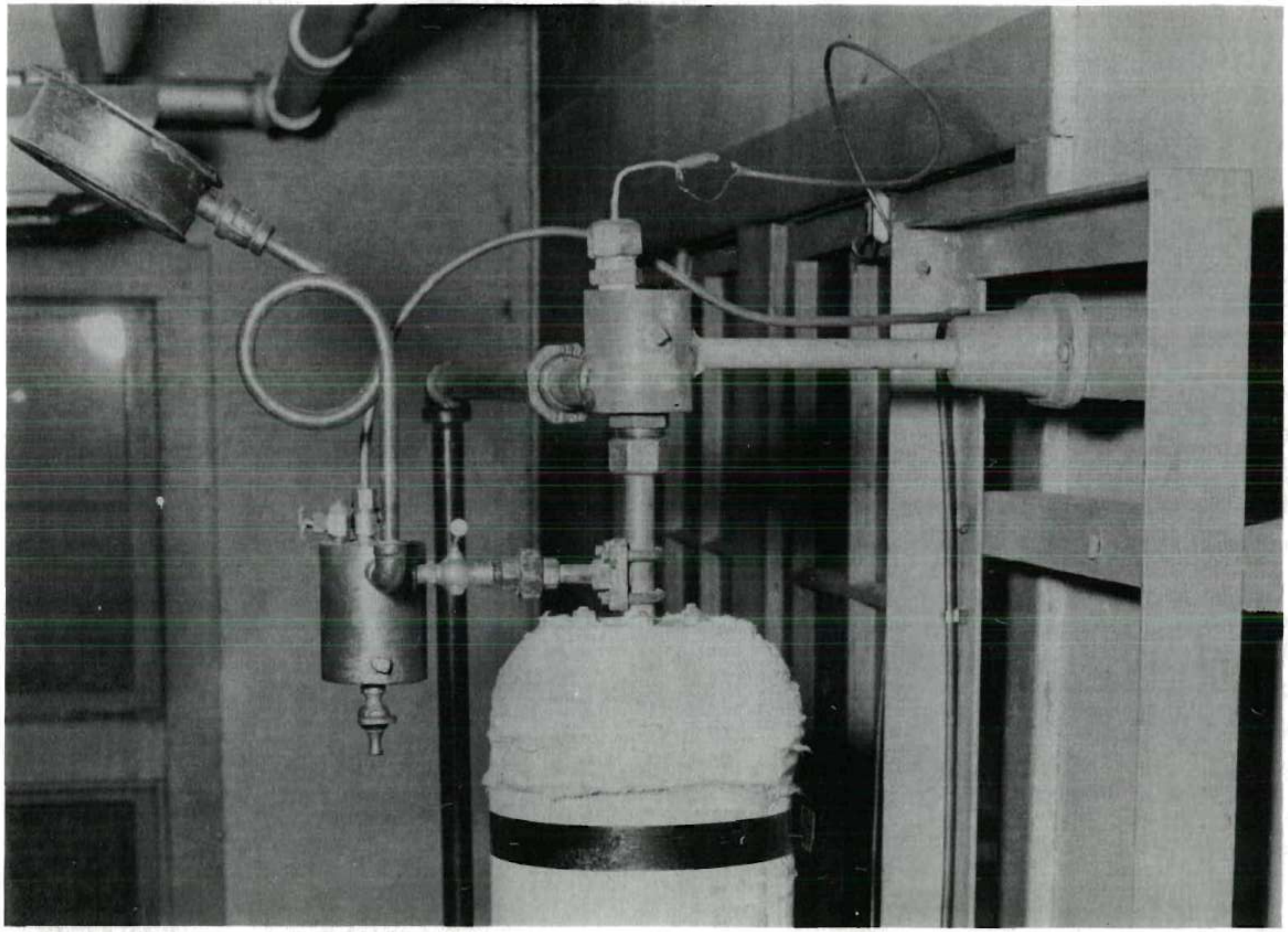


Figure 11. Upper Portion of Heat Transfer Apparatus Before the Complete Addition of Insulation.

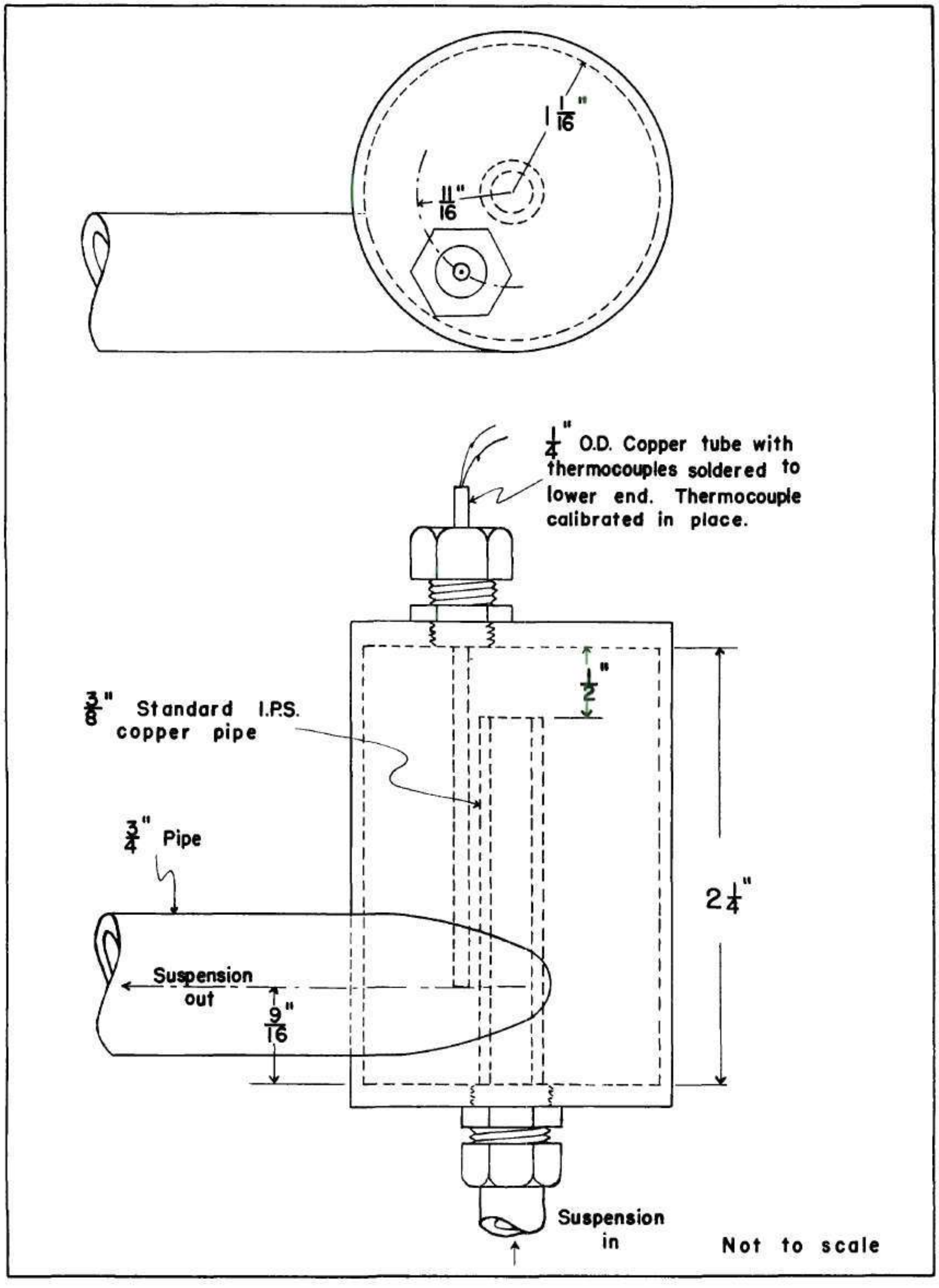


Figure 12. Details of Construction of the Mixing Chamber Permitting the Determination of Bulk Suspension Temperature.

be regulated. A thermocouple mounted in the end of a small copper tube and a calibrated thermometer were also located in the tank, providing not only a means of temperature measurement but also a ready means by which the thermocouple system could be checked.

The heat transfer fluid was withdrawn from the bottom of the mixing tank by a 3/4-inch, close-coupled, side-suction pump, having an enclosed type impeller, and driven at 3,500 rpm by a 1/4-HP motor. The pump was manufactured by the Aurora Pump Company, Aurora, Illinois. The stream leaving the pump could be diverted to return to the tank directly or could pass through the heat exchange system.

That portion of fluid to pass through the system went directly from the pump through a 1-1/4-inch rotameter and then to the straightening section described above. The rotameter was the Size No. 7 Universal Model of the Schutte and Koerting Company, Philadelphia, Pennsylvania. A stainless steel float, identical in shape to those commonly employed in such a device, was used. It differed by having angularly milled grooves about the periphery of the portion of major diameter for the purpose of causing rotation. In pure liquids and in dilute suspensions the rotameter was useful in establishing flow of the desired magnitude and noting the constancy of flow; in heavy suspensions the position of the float was quite undetectable even with an intense light.

The drop in pressure accompanying flow through the heat exchange section was measured with a mercury-filled manometer. Because of constructional limitations the pressure taps were located a little more than one inch outside the actual heat exchange section. Details of the method of attaching the taps to the copper pipe are shown by the exploded drawing of Figure 13. Figure 11 shows the actual installation at the upper end of the exchange section, while the lower tap may be seen in Figure 1.

The steam required was taken from an overhead line. As may be seen in Figure 2, the steam passed first through a steam separator. The separator, shown in Figure 14, followed the design of commercial models but was specially constructed.

On the same line but more than six feet below the steam separator a steam calorimeter was located. The design of both the calorimeter and the sampling tube which was inserted in the steam line faithfully followed that recommended in Power Test Codes, ASME Series 1929, Instruments and Apparatus, Part II. The calorimeter temperature was measured with an ASTM-certified thermometer inserted the prescribed depth into the oil-filled thermometer well of the calorimeter.

Since the pressure in the main stream line averaged about 150 lb./in.^{2} gauge, it was necessary to reduce and regulate the pressure in the heat exchange section. For this two 1/2-inch, No. 960 regulators of the Crane Company, Chicago, Illinois, were used. One regulator supplied steam at pressures of from 5 to 30 pounds, the other at pressures of from 20 to 100 pounds.

As discussed in regard to the design of the heat exchange section, two streams of condensate, one of which originated with the steam condensing on the copper pipe and the other with the steam condensing on the outside wall, were withdrawn from the section. To maintain a certain

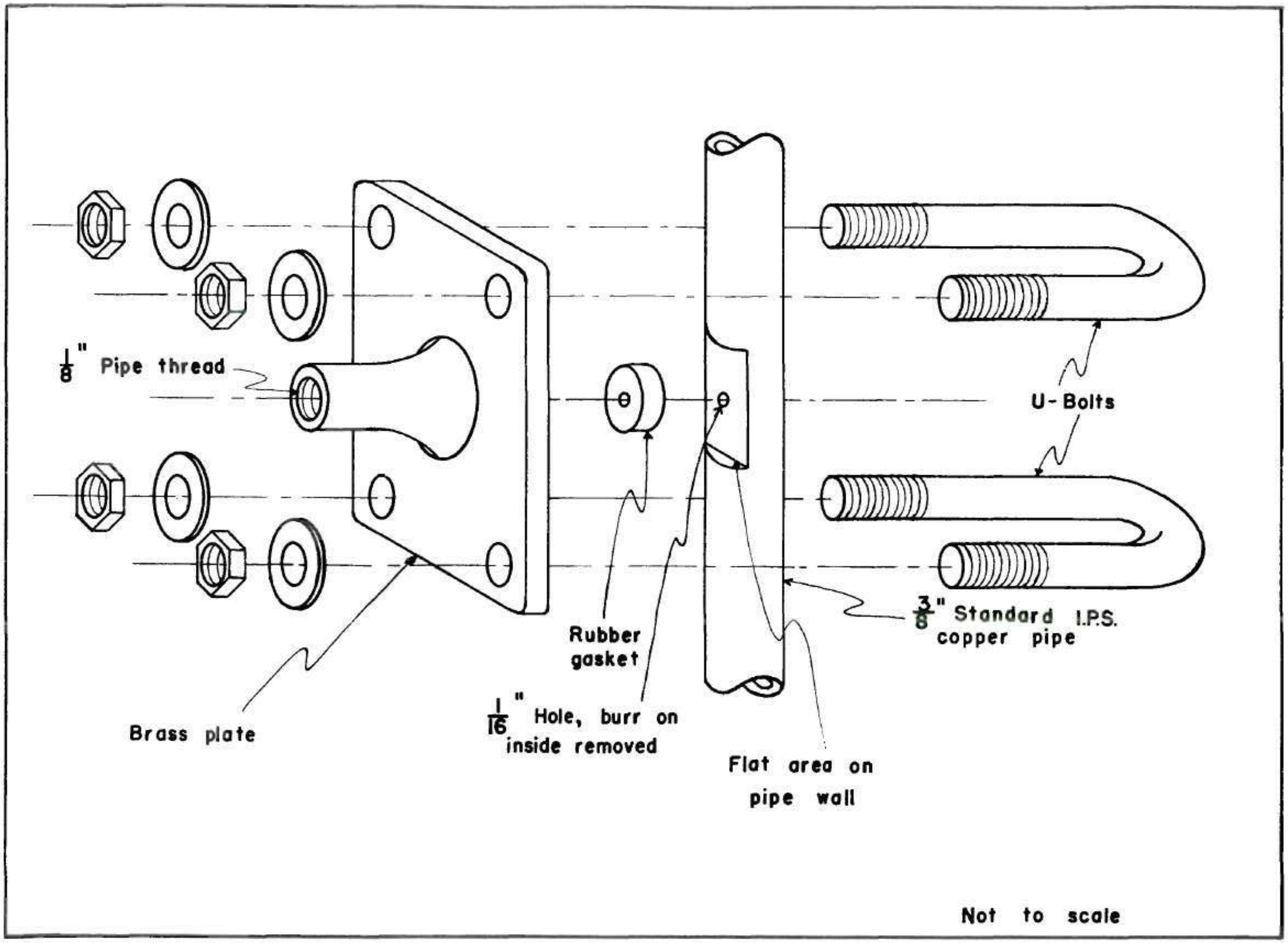


Figure 13. Detail of Means of Attaching Pressure Tap to Pipe.

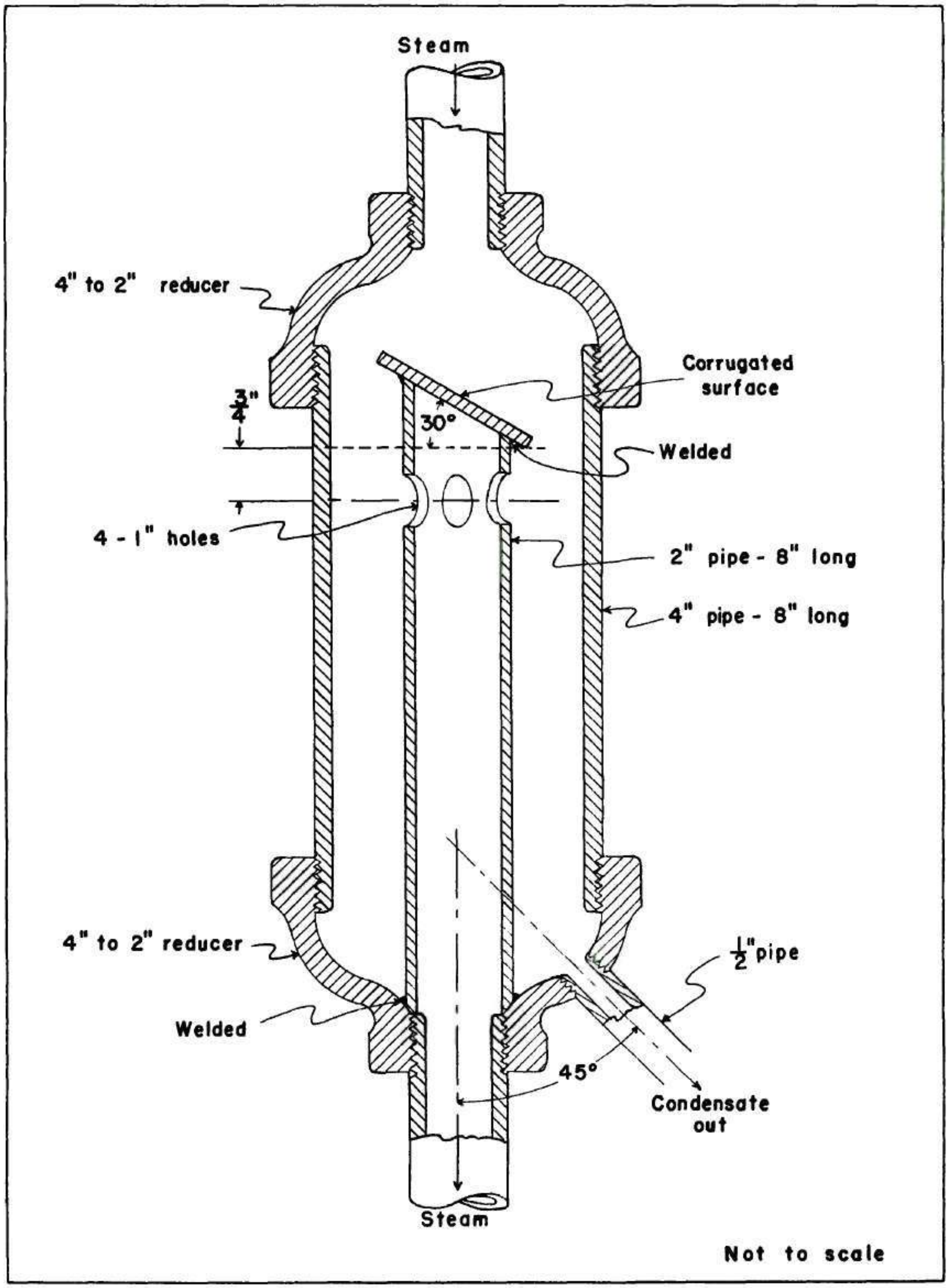


Figure 14. Details of Construction of Steam Separator.

pressure in the section, therefore, a steam trap or another system was required on each exit line. No. 60, 1/2-inch Yarway impulse traps, manufactured by the Yarnall-Waring Company, Philadelphia, Pennsylvania, were used. In addition, a valve arrangement permitted the condensate coming from the inner chamber, i.e., from the copper pipe surface, to be discharged either through the stream trap or through a measuring system. The measuring system consisted of a sight glass, by means of which the quantity of this condensate in the system could be viewed, and a Size No. 2 or 3 Universal Model, Schutte and Koerting Company, Philadelphia, Pennsylvania, rotameter. The rotameters were calibrated over their entire range. With this arrangement the flow of condensate through the rotameter could be regulated so that there was neither an increase nor a decrease in the system. A thermocouple was installed within this condensate line so that the condensate's temperature might be determined. Provision was also made so that the condensate from the outer chamber of the heat transfer section could either be collected and measured or discharged down the drain.

All steam lines were insulated with the appropriate size of 85 per cent magnesia pipe insulation.

As mentioned in the preceding discussions, sixteen thermocouples were installed with which to measure the wall temperature of the copper pipe: one each to measure the inlet and outlet fluid temperature, one to measure the fluid temperature in the mixing tank, and one to measure the leaving condensate temperature. From these diverse points, leads ran to a No. 820, twenty point thermocouple selector switch manufactured by the Wheelco Instruments Company, Chicago, Illinois. Determination was made with the type K-2 potentiometer of the Leeds and Northrup Company, Philadelphia, Pennsylvania, using also a Leeds and Northrup enclosed lamp and scale type of galvanometer having a sensitivity of 25 microvolts per division of the attached scale. The low temperature coefficient form of standard cell manufactured by the Eppley Laboratory, Inc., Newport, Rhode Island, was employed. The cell's voltage was checked against that of two other cells before use and was checked again near the midpoint of the investigation; the voltage was found to correspond to the manufacturer's rating each time. Thermocouples, after being installed in the pipe wall, could not be calibrated. However, a thermocouple installed in a short segment of pipe, the thermocouples mounted in tubes for insertion into the fluid or condensate streams and unmounted thermocouples were calibrated. No significant differences in the temperature response of these thermocouples were found; the calibration data are given in the Appendix. Calibration was accomplished by using the potentiometric arrangement previously described with the exception that a Leeds and Northrup galvanometer arranged to have a sensitivity of 0.32 microvolts per millimeter of scale was employed. The thermocouples were compared with thermometers calibrated to 0.1 degree Fahrenheit by the U. S. Bureau of Standards. In accordance with the procedure outlined by Roeser and Wensel (1941), comparisons were made at three temperatures, 111.2° F., 211.3° F. and 321.4° F.; using the average value of the potential indicated at each temperature, the results were expressed by an equation of the form, $\text{emf} = aT + bT^2 + cT^3$, and interpolation was made using this equation.

Roeser and Wensel state that this procedure may be expected to "give interpolated values as accurately as the couple can be relied upon to retain its calibration."

As may be seen in Figure 1, Bourdon gauges were employed to measure steam pressure as well as to measure the pressure on the fluid. The location of the gauge to indicate the pressure on the fluid is clearly shown in Figure 11; its sole purpose was to insure that the pressure on the fluid in the heat exchange section was sufficient to prevent boiling. Gauges to indicate steam pressures were located on the entering line near the calorimeter, on the line carrying the reduced-pressure steam to the heat exchange section, on the condensate exhaust line and, for the latter runs, on the air vent from the heat exchange section. All gauges were calibrated with a dead-weight gauge tester.

Valves and cocks were employed at various points to control the flow of the heat transfer fluid, steam or cooling water and to permit draining and cleaning the system.

Other special devices were prepared to measure other quantities related to the investigation; the appropriate apparatus are described in the section devoted to these measurements.

2. Experimental Procedure

It was found expedient to accomplish several runs in one period of operation, then to drain and flush the system, and not to attempt other runs until another period of sufficient duration was assured to enable completion of several more runs. Therefore, at the beginning of each set of runs the first step was always the partial filling of the mixing and storage tank with pure liquid (usually water; in the latter runs, ethylene glycol). After the tank was filled to the desired level (about half full), the stirrer in the tank and the circulating pump were started. The latter, by proper adjustment of the valves, forced the liquid at a moderate rate through the heat transfer section, through the cooler and back into the tank. Next, the steam line was freed of condensate, and the drain from the steam separator was opened so that future condensate from the line would also go to the drain. The valves which controlled the flow of condensate from the heat exchange section were adjusted so that exhaust could occur only through the steam traps, the vent which permitted the escape of noncondensable gases was opened, the flow of water to the heat exchanger which served to cool the liquid was started, and the calorimeter was opened to the steam line. As the final step in the initial preparation, steam was admitted to the heat exchange section.

The apparatus was allowed to operate for 15 to 20 minutes at conditions such that the liquid in the mixing tank came nearly to the boiling temperature. After this period, by adjusting flow rates or the steam pressure, the liquid's temperature was brought to the desired operating range. The purpose of the higher temperature was to drive absorbed gases from the liquid.

At this point, dry, powdered material might be added to the liquid to form the suspension to be tested, though on many occasions the first run was made with pure liquid so that the functioning of the apparatus could be assured. The procedure followed when a suspension was employed instead of the liquid differed only in the sampling of the suspension and in the determining of the flow rate by direct weighing instead of with the rotameter, so the remaining discussion will presuppose the use of a suspension. After thorough mixing of the solid material with the liquid, the cocks exposing the mercury manometer to the pressure in the system and the Bourdon gauge to the pressure at the upper end of the heat exchange section were opened. The suspension flow was adjusted at this point to give conditions of pressure and rate near those desired.

After an initial period of equilibration to permit lines, valves, etc., to adjust to the new temperature conditions, the steam pressure in the exchange section was readjusted to a predetermined value, the suspension flow rate through the exchange section was likewise reset and, if the temperature of the suspension going to or from the heat exchange section was desired at another level, the cooling water flow rate to the liquid cooler was adjusted accordingly. The condensate produced in the chamber immediately surrounding the copper pipe, instead of exhausting through a steam trap, was now routed through the sight glass and rotameter arrangement; the controlling valve was set so that the level of the condensate was visible and as nearly constant as possible. Now began a period, usually lasting several hours, during which temperatures were occasionally checked, and the condensate flow controlling valve was occasionally adjusted and which ended when equilibrium (as indicated by temperature constancy) was found to have been established.

Upon the attainment of equilibrium, the temperatures along the pipe wall, the temperatures of the suspension entering and leaving the heat exchange section, the condensate temperature, the suspension temperature in the mixing tank, the calorimeter temperature, the pressure drop through the exchange section, the pressure at the top of the heat exchange section, the steam pressures in the main line and in the heat exchange section, the barometric pressure and the rate of steam condensation were recorded. The suspension flow rate was obtained by diverting its flow from the mixing tank into another container for a certain time interval. The weight of suspension so diverted was obtained, a sample was taken and the remaining suspension was returned to the tank. All readings were recorded several times to minimize the effect of a certain fluctuation in most measured values; the average result was used in the calculations. From the sample the suspension concentration was obtained by pycnometer weighings usually at a later date.

After the completion of a run more powdered solid might be added to the suspension, any or all other conditions might be changed or the experiments might be stopped. In the latter event, the procedure followed was essentially the reverse of that described previously, i.e., the steam, the cooling water and the suspension flow were stopped in the order named.

In certain runs another test was made, and in others special conditions were encountered. The test involved a determination of the rate of steam condensation in the outer jacket of the heat transfer section. This was accomplished by diverting the condensate stream from its normal course to the drain into a container of ice and ice water and determining the rate by weight change. The special condition concerned the dispersion of the solid material, which, in the case of the aluminum powder and water, required the addition of not more than 0.02 per cent by weight of the commercial wetting agent, Aerosol O. T. of Stansi Scientific Supplies, Chicago, Illinois. (The Proctor and Gamble product "Tide" was later found to be a better dispersing agent.) The addition of such a small amount of agent could have had no direct effect on heat transfer properties but its indirect effect, exerted through its action on viscosity, etc., may have been considerable. Sampling of the suspension as described above for other determinations tended to insure coordinated results, however.

3. Data

Descriptive and experimental heat transfer data for all runs are given in Tables I and II, respectively, and typical data sheets upon which experimental results were recorded are presented on pages 159 and 160 in the Appendix. It will be noted that, with the exception of the one run employed in making the sample calculation, individual temperatures along the pipe wall are not given, and that only temperatures representing conditions at each extremity of the heated portion of the copper pipe are shown. This is permissible since in every case the temperature was found to vary linearly between these limits. Experimental data for one run confirming this assumption are presented in the following section.

4. Sample Calculation

Run No. 32, employing a suspension of water and glass beads, will be used in the sample calculation presentation. This run is chosen because it is typical and because the results are generally good. In addition to the data given in Table I and on the data sheets in the

TABLE I

GENERAL FLUID SYSTEM INFORMATION

Run No.	Fluid System	Weight Per Cent Solid Material
1	Water only	0
2	Water and attapulgus clay	0.79
3	"	3.34
4	"	5.23
5	"	7.18
6	Water only	0
7	"	0
8	"	0
9	Water and powdered copper	0.71
10	"	0.71
11	"	0.71
12	"	0.83
13	"	0.83
14	"	0.83
15	"	1.26
16	"	1.26
17	"	14.6
18	"	14.2
19	"	13.7
20	"	13.7
21	"	13.7
22	"	25.8
23	"	25.8
24	"	14.4
25	"	34.3
26	"	36.0
27	"	39.2
28	"	38.6
29	Water only	0
30	Water and No. 18 glass beads	8.39
31	"	5.98
32	"	16.5
33	"	17.2
34	"	14.5
35	"	25.1
36	"	23.6
37	Water and No. 9 glass beads	23.9
38	"	21.6
39	"	43.9
40	"	34.5
41	"	45.7

(Continued)

TABLE I (Concluded)

GENERAL FLUID SYSTEM INFORMATION

Run No.	Fluid System	Weight Per Cent Solid Material
42	Water only	0
43	Water and powdered graphite	3.70
44	"	9.01
45	"	9.01
46	"	9.01
47	"	13.3
48	"	13.4
49	"	13.4
50	"	16.5
51	"	16.5
52	"	19.4
53	"	19.4
54	"	24.7
55	"	24.7
56	"	29.1
57	"	29.1
58	"	31.8
59	"	31.8
60	Water only	0
61	Water and powdered aluminum	4.01
62	"	4.01
63	"	5.57
64	"	5.57
65	"	9.90
66	"	9.90
67	"	9.03
68	"	9.03
69	"	13.4
70	"	13.4
71*	Water only	0
72**	"	0
73	"	0
74	Ethylene glycol only	0
75	Ethylene glycol and powdered graphite	11.1
76	"	17.1
77	"	17.1
78	Ethylene glycol and powdered aluminum	9.12
79	"	14.7

*Duplication of run No. 1
**Duplication of run No. 42

TABLE II

EXPERIMENTAL HEAT TRANSFER DATA

Run No.	Rate of Fluid Flow (lb./min.)	Fluid Pressure Drop in Exchange Section (in. Hg)	Minimum Pressure on Fluid (psia)	Main Steam Line Pressure (psia)	Barometric Pressure (in. Hg)	Calorimeter Temperature (°F)	Steam Condensed (lb./min.)	Approximate Steam Pressure in Exchange Section (psia)	Temperature of Leaving Condensate (°F)	Pipe Wall Center Temperature* Inlet (°F)	Pipe Wall Center Temperature* Outlet (°F)	Fluid Temperature Inlet (°F)	Fluid Temperature Outlet (°F)
1	58.0	12.78	--	134	29.1	263.8	3.51	25.6	228.5	227.0	234.0	137.5	197.0
2	43.9	10.45	--	134	29.0	278.4	3.02	27.7	230.8	231.1	232.1	135.6	204.1
3	42.8	10.67	--	133	29.2	279.0	3.02	27.1	229.1	231.3	231.9	132.8	204.3
4	43.7	10.76	--	136	29.0	278.6	2.93	28.0	230.5	231.1	231.8	136.1	207.3
5	43.8	10.89	--	135	29.0	278.6	2.59	26.2	224.0	231.4	231.6	140.4	205.3
6	69.3	14.99	--	135	29.1	284.9	3.77	23.2	228.0	223.9	232.8	143.7	196.8
7	41.9	10.36	--	134	29.2	284.5	2.78	22.0	227.9	217.0	226.0	121.3	187.2
8	69.3	15.31	--	135	29.0	284.3	3.44	16.7	216.0	204.0	212.5	128.1	177.4
9	71.3	15.84	--	134	29.0	285.8	3.65	18.6	219.3	206.6	215.0	130.3	181.6
10	34.2	9.40	--	136	29.0	284.9	2.37	17.9	218.1	210.2	216.9	110.6	181.0
11	67.7	14.89	--	134	29.0	284.9	4.31	28.3	232.4	232.3	234.1	137.2	198.3
12	28.8	8.70	17.3	135	29.0	283.1	2.61	30.0	231.0	230.7	231.8	113.8	204.1
13	48.7	11.32	17.3	136	29.0	283.1	3.36	26.2	223.3	229.4	231.4	126.1	196.7
14	79.5	17.67	15.8	136	29.0	283.8	3.62	25.2	230.9	227.8	232.7	156.0	203.1
15	81.2	18.08	15.8	135	29.0	284.0	3.65	25.8	229.2	226.3	233.2	158.6	205.4
16	46.8	11.29	15.8	135	29.0	284.9	2.76	19.1	216.8	213.0	224.9	129.0	188.7
17	85.5	18.33	17.4	133	29.3	281.7	3.99	26.7	232.7	229.5	234.3	150.8	204.2
18	53.5	12.51	15.9	133	29.3	285.8	2.81	29.0	233.9	233.9	234.0	154.7	211.8
19	90.8	19.85	15.9	131	29.3	279.5	3.26	21.8	229.1	218.0	227.4	159.0	199.2
20	90.8	18.69	16.9	133	29.3	276.8	4.05	32.5	234.9	232.4	234.9	163.6	215.6
21	76.7	16.36	17.3	132	29.1	282.6	3.75	30.7	232.4	232.3	232.8	156.8	210.5
22	83.6	17.20	17.3	132	29.1	283.1	3.84	32.9	234.4	232.1	232.7	159.9	216.9
23	83.6	17.86	17.3	136	29.1	279.5	2.86	17.1	210.2	209.5	214.9	144.1	190.3
24	48.5	11.48	24.3	136	29.1	282.2	2.36	36.4	251.3	250.0	251.8	178.4	230.8
25	110.0	21.91	16.4	160	29.3	292.1	3.65	29.6	231.1	231.0	231.4	164.4	210.6
26	65.9	15.52	17.4	160	29.3	291.6	2.52	20.1	221.2	217.9	226.2	130.3	194.1
27	116.0	23.48	17.4	158	29.3	292.1	3.66	28.0	231.6	230.3	231.8	156.6	205.9
28	34.7	12.27	17.8	132	28.8	283.6	1.47	20.3	226.3	214.5	217.7	132.9	199.9
29	39.9	10.14	15.8	161	29.1	291.6	2.26	17.1	216.1	209.6	218.0	124.9	182.2
30	40.6	10.80	17.1	160	29.1	292.1	2.42	17.1	216.7	211.1	217.3	125.3	188.9
31	75.2	17.08	16.4	161	29.4	291.9	3.51	27.3	231.7	228.0	232.7	156.0	204.1
32	79.4	18.00	16.4	161	29.4	292.4	3.51	28.8	231.6	231.2	232.2	156.6	206.2
33	48.5	12.09	16.9	160	29.4	293.0	2.73	25.1	321.7	230.8	232.0	134.1	200.6
34	27.2	9.15	17.4	162	29.4	292.8	1.77	25.2	231.6	230.8	232.6	129.3	205.2
35	84.2	19.92	15.3	161	29.2	291.2	3.30	25.2	235.2	229.9	239.2	148.7	199.5
36	55.9	13.64	16.8	162	29.2	291.9	2.66	19.5	214.8	217.3	222.5	126.1	187.2
37	80.0	16.75	16.3	159	29.1	289.8	2.97	24.7	230.8	227.3	234.1	159.4	204.7
38	48.0	11.35	17.8	161	29.2	290.3	2.38	27.8	241.3	233.2	242.7	152.3	210.7
39	84.3	18.28	16.3	160	29.1	290.3	3.07	26.1	232.0	229.6	237.6	149.9	202.7
40	40.9	11.90	17.3	160	29.1	271.8	2.04	32.6	253.8	247.2	254.1	145.9	215.9

(Continued)

TABLE II (Concluded)

EXPERIMENTAL HEAT TRANSFER DATA

Run No.	Rate of Fluid Flow (lb./min.)	Fluid Pressure Drop in Exchange Section (in. Hg.)	Minimum Pressure on Fluid (psia)	Main Steam Line Pressure (psia)	Barometric Pressure (in. Hg)	Calorimeter Temperature (°F)	Steam Condensed (lb./min.)	Approximate Steam Pressure in Exchange Section (psia)	Temperature of Leaving Condensate (°F)	Pipe Wall Center Temperature* Inlet (°F)	Pipe Wall Center Temperature* Outlet (°F)	Fluid Temperature Inlet (°F)	Fluid Temperature Outlet (°F)
41	84.1	17.80	16.8	159	29.1	290.3	1.68	23.2	231.5	221.7	232.0	178.2	211.2
42	44.0	10.87	17.5	162	29.5	290.3	2.46	20.1	226.4	220,5	226.9	136.0	191.7
43	46.4	11.23	17.5	161	29.4	291.2	2.48	19.7	219.5	219.6	226.0	136.2	191.8
44	42.7	10.72	18.4	159	29.2	284.9	1.95	24.3	228.2	225.9	228.2	155.7	203.2
45	70.7	15.37	17.9	160	29.2	290.3	2.25	23.4	222.1	225.2	228.6	172.4	205.8
46	71.5	15.51	17.9	159	29.2	289.8	--	26.4	230.9	222.8	230.5	142.7	194.8
47	73.7	15.81	17.9	160	29.2	290.7	3.78	27.0	231.1	226.3	232.7	143.3	199.1
48	45.5	11.62	17.9	161	29.3	291.2	2.26	23.1	230.0	226.8	233.2	145.3	202.7
49	27.8	9.36	17.9	157	29.3	291.2	1.75	19.8	214.3	216.0	221.6	114.0	190.5
50	82.0	17.15	16.6	162	29.3	291.6	4.24	33.8	234.3	231.2	232.5	154.6	210.5
51	84.7	18.00	15.9	160	29.3	291.2	2.30	23.2	231.2	213.6	232.5	172.0	203.9
52	58.4	12.89	16.4	158	29.3	290.7	2.34	24.0	230.3	226.5	232.3	157.1	205.8
53	31.1	9.73	17.4	159	29.3	290.5	1.39	20.2	214.5	214.0	222.6	141.7	198.7
54	63.6	14.62	16.4	163	29.3	290.3	2.07	20.6	221.3	211.3	227.4	157.4	200.3
55	79.5	--	17.4	160	29.3	290.8	3.74	35.1	237.5	231.9	237.1	160.8	214.7
56	77.9	16.37	16.8	157	29.1	290.3	2.69	25.3	232.0	226.2	233.6	159.2	203.7
57	40.6	10.81	16.8	162	29.1	291.2	1.57	19.3	221.7	212.1	227.2	124.7	172.1
58	77.1	18.09	16.8	157	29.1	290.3	2.33	22.8	228.2	220.3	228.0	154.7	196.3
59	45.1	11.76	16.8	158	29.1	229.9	1.62	23.5	227.9	220.5	224.9	127.4	172.1
60	28.9	8.69	16.8	161	29.0	290.7	1.52	20.4	213.7	209.7	212.7	143.7	197.7
61	56.5	12.58	16.3	163	29.0	290.3	2.36	22.5	229.7	224.8	232.3	155.7	199.0
62	71.0	16.48	16.3	159	29.0	290.7	3.74	30.2	244.5	233.1	243.2	152.0	202.7
63	26.3	8.40	17.3	160	29.1	291.4	1.70	22.1	214.4	209.3	213.4	123.6	194.5
64	77.7	--	19.3	163	29.1	291.9	3.03	20.1	220.3	209.3	221.3	148.3	188.8
65	75.1	--	--	161	29.1	291.0	2.98	23.0	231.4	222.4	227.9	150.4	192.4
66	48.1	--	--	160	29.1	291.2	2.22	20.4	223.8	213.7	225.0	134.3	184.3
67	71.3	--	--	161	29.3	291.2	2.58	20.4	221.5	214.6	224.8	149.7	188.9
68	37.3	--	--	160	29.2	291.6	1.72	19.9	222.3	219.1	225.8	135.0	188.4
69	69.6	--	--	160	29.3	291.0	2.30	19.8	223.3	211.2	224.7	148.1	186.2
70	34.4	--	--	166	29.3	291.7	1.44	20.3	223.6	221.6	226.6	135.7	185.5
71	56.8	--	--	164	29.4	291.2	2.99	26.3	230.7	224.0	232.6	146.3	196.0
72	43.9	--	--	163	29.4	291.2	2.51	20.3	218.3	211.8	225.2	134.6	188.4
73	15.4	--	--	164	29.4	292.1	1.37	28.6	230.8	232.0	235.2	106.3	199.2
74	55.7	17.30	17.1	164	28.8	286.3	0.64	19.6	224.2	223.0	224.1	140.8	160.2
75	58.2	18.31	17.4	159	29.2	286.7	0.80	22.3	232.1	231.0	233.2	144.3	169.1
76	58.8	18.85	17.7	162	29.2	287.8	0.77	22.3	235.6	231.7	236.3	143.9	166.1
77	40.9	13.15	16.8	163	29.2	285.3	0.45	23.2	237.7	237.0	237.0	136.5	158.4
78	42.1	13.15	16.8	161	29.0	286.7	0.42	22.4	227.6	228.6	230.8	152.3	171.5
79	42.8	14.12	16.3	162	29.0	286.7	0.35	22.2	231.9	272.9	234.3	161.3	177.4

*Obtained by extrapolation of best line through pipe wall temperature data.

Appendix, pages 159 and 160, other values determinable from the geometry of the heat exchange apparatus are needed in the calculation.

Heat was transferred through a nominal 3/8-inch, standard iron pipe size copper pipe for a length of 7.70 feet. Pressure drop measurements were made over a slightly longer distance, 7.95 feet. The pipe had an inside diameter of 0.0411 feet and an outside diameter of 0.0563 feet; its total inside heat transfer surface was 0.994 square feet, and its outside heat transfer surface was 1.361 square feet. The log mean heat transfer surface area was therefore 1.164 square feet. The pipe had an internal cross-sectional area of 0.00133 square feet; the pipe wall thickness was 0.00758 feet.

Data relating to the physical properties of the materials required in this and other calculations are given in the Appendix. The source of the data is also given there.

Using the nomenclature and data of Keenan and Keyes (1936), h_2 is the enthalpy of superheated steam at the calorimeter pressure and temperature, h_f is the enthalpy of the liquid in the high-pressure steam entering the calorimeter, and h_{fg} is the latent heat of vaporization of the high-pressure steam; then the quality of the steam, x, entering the heat exchange apparatus is

$$x = \frac{h_2 - h_f}{h_{fg}} = \frac{1188.7 - 336.4}{858.8} = 0.993 \text{ or } 99.3\ \%. \qquad (3)$$

In obtaining a heat balance, the rate of heat entering the heat exchange section was determined as the product of the quantity of steam condensed in unit time and the enthalpy of the entering steam less the enthalpy of the leaving liquid. The enthalpy of the steam entering the exchange section was considered the same as that of the steam in the calorimeter; the enthalpy of the leaving liquid condensate was determined from its temperature. Thus, using data for steam given by Keenan and Keyes, the heat entering was at the rate of

$$3.51 \frac{\text{lb-mass}}{\text{min.}} \times (1188.7 - 199.8) \frac{\text{Btu}}{\text{lb-mass}} = 3470 \frac{\text{Btu}}{\text{min.}} .$$

The rate of heat leaving the system was determined as the product of the suspension flow rate, the change in suspension temperature and the suspension heat capacity. This last quantity was taken as the sum of the heat capacities on a weight basis of the liquid and of the solid at bulk mean suspension temperature, 181.4° F., each multiplied by its respective weight fraction in the suspension. Therefore, using heat capacity data given in the Appendix, the suspension heat capacity was calculated to be

$$C_p = (0.165 \times 0.180) + (0.835 \times 1.004) = 0.868 \frac{\text{Btu}}{\text{lb-mass, }^\circ\text{F.}} ,$$

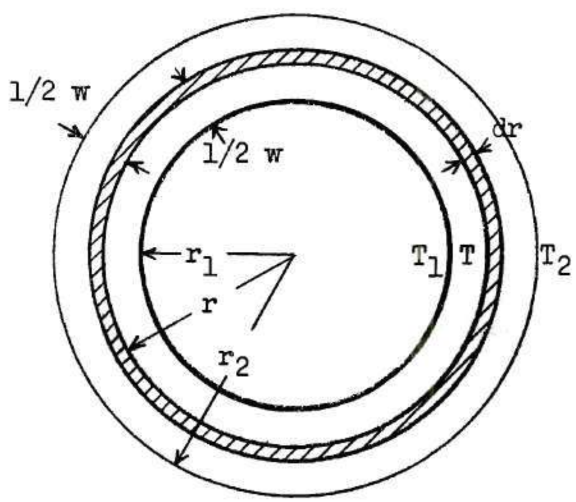
and the heat leaving the system was found to be at the rate of

$$79.4 \frac{\text{lb-mass}}{\text{min.}} \times (206.2 - 156.6)\ ^\circ\text{F.} \times 0.868 \frac{\text{Btu}}{\text{lb-mass, }^\circ\text{F.}}$$

$$= 3420 \frac{\text{Btu}}{\text{min.}} .$$

Assuming the heat entering the system to be correct, this value, 3420 Btu/min., is in error by $\frac{3470 - 3420}{3470} \times 100 = 1.4$ per cent. The heat entering is believed the more reliable measure of the heat transferred because a calculation of heat capacity is not required; it was used in subsequent calculations for all runs with the exception of No. 46. The rate of steam condensation was not recorded for this run.

For a pipe having the dimensions stated above and with thermocouples embedded at the center of the pipe wall, it may be shown that 54 per cent of the temperature drop across the pipe wall occurs from the plane of the thermocouples inward. To show that this is so, it is necessary to obtain an expression for the temperature distribution through the pipe wall when heat is flowing through the wall at a steady rate. The necessary relationships may be arrived at as follows: For a long pipe, let T_1, T and T_2 be the temperatures at the radial distances r_1, r and r_2, respectively (see the accompanying sketch). When the pipe has length L, the heat transmitted in unit time, q, if k is the thermal conductivity of the pipe material, is given by the relationship



$$q = 2kL\pi r\frac{dT}{dr} . \qquad (4)$$

Integration of equation 4 between the limits of r_2 and r_1 and between T_2 and T_1 gives

$$q = 2kL\pi \frac{T_2 - T_1}{\ln r_2 - \ln r_1} , \qquad (5)$$

and integration of equation 4 between the limits of r and r_1 and between T and T_1 gives

$$q = 2kL\pi \frac{T - T_1}{\ln r - \ln r_1} \; . \tag{6}$$

Since $q = q$, equations 5 and 6 may be solved simultaneously giving

$$\frac{T - T_1}{T_2 - T_1} = \frac{\ln r - \ln r_1}{\ln r_2 - \ln r_1} \tag{7}$$

Values for $\ln r_1$, $\ln r$ and $\ln r_2$, when evaluated from the pipe dimensions and substituted into equation (7), give

$$\frac{T - T_1}{T_2 - T_1} = \frac{\ln (0.02435) - \ln (0.02055)}{\ln (0.02815) - \ln (0.02055)} = 0.5379 \; .$$

Therefore, the actual inside pipe surface temperature must be calculated from the midpoint, wall temperature measurements. As shown in Figure 15, a plot of the measured center pipe wall and bulk suspension temperatures, the wall temperature is well represented by a straight line; since the thermal properties of the suspension change very slightly with temperature, the bulk mean suspension temperature may also be represented by a straight line. While it is obvious from the figure that some segments of the pipe are transferring more heat than others, the mean temperature drop occurring across the pipe wall may be calculated from the dimensions of the pipe, the total heat flowing and the pipe wall thermal conductivity. Thus, the total mean temperature drop across the pipe wall was found to be

$$3570 \times 60 \frac{\text{Btu}}{\text{hr.}} \times 0.00758 \text{ ft. in thickness} \times \frac{1 \text{ hr., ft.}^2, {}^{\circ}\text{F.}}{217.6 \text{ Btu, ft.}}$$

$$\times \frac{1}{1.164 \text{ ft.}^2 \text{ of heat transfer area}} = 6.2^{\circ} \text{ F.}$$

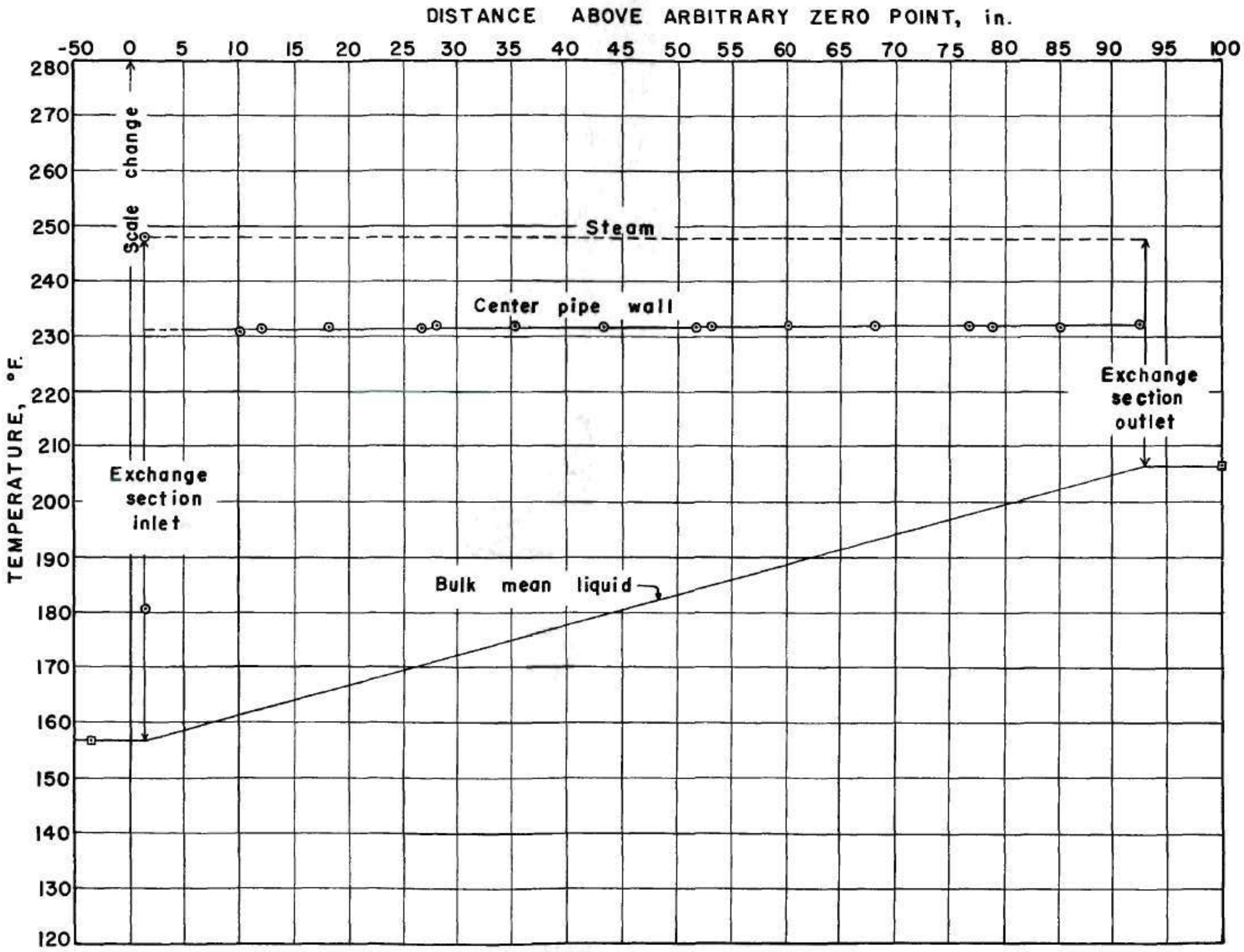


Figure 15. Temperatures Established in Heat Exchange Section for Run No. 32.

The temperature drop from the plane of temperature measurement to the inside surface of the pipe was then only

$$6.2^{\circ}\ F. \times 0.54 = 3.4^{\circ}\ F.$$

Referring again to Figure 15, it may be seen that the mean temperature drop through which heat was transferred from the pipe wall to the suspension will be the logarithmic mean calculated from the inside pipe surface temperatures and the bulk mean temperature of the suspension all evaluated at the points of entrance into and exit from the heat transfer section. Thus,

$$\Delta T_m = \frac{\Delta T_i - \Delta T_o}{\ln \frac{\Delta T_i}{\Delta T_o}} = \frac{[(231.2 - 3.4) - 156.6] - [(232.2 - 3.4) - 206.2]}{2.303 \log \frac{[(231.2 - 3.4) - 156.6]}{[(232.2 - 3.4) - 206.2]}}$$

$$= 42.4^{\circ}\ F.$$

The average coefficient of heat transfer between the pipe wall and the suspension was calculated directly from the rate of heat transfer, the mean temperature drop and the area across which the heat flowed. Therefore,

$$h = 3470 \times 60\ \frac{Btu}{hr.} \times \frac{1}{0.994\ ft.^2} \times \frac{1}{42.4^{\circ}\ F.} = 4940\ \frac{Btu}{hr., ft.^2, {}^{\circ}F.}$$

The coefficient of cubical expansion of a solid is generally much less than that of a liquid. In the case of the system water-glass beads, the change in the density of glass with temperature over the temperature range and in the concentration involved is quite insignificant in calculating the suspension density at conditions other than the conditions of measurement, i.e., room temperature. Therefore, the suspension density at the experimental conditions was calculated as though the change with temperature were due entirely to the liquid. The acceleration of gravity at the place of measurement was 32.14 ft./sec.^{2}, according to Hodgman (1946), and since a vertical distance of 7.95 ft. existed between the points of pressure drop measurement and the mean suspension bulk temperature was such that the water had a density of 0.970 gm./cc. instead of 0.997 gm./cc., as it had at 27.4° C., the static pressure due to the suspension head was

$$7.95 \text{ ft.} \times 69.9 \frac{\text{lb-mass}}{\text{ft.}^3} \times \frac{0.970}{0.997} \times \frac{\text{ft.}^2}{144 \text{ in.}^2} \times$$

$$\frac{32.14 \text{ lb-force, ft., sec.}^2}{32.17 \text{ lb-mass, sec.}^2\text{, ft.}} = 3.75 \frac{\text{lb-force}}{\text{in.}^2} .$$

The pressure drop due to friction, P_f, was therefore

$$\left[(18.00 \times 0.4911) - 3.75\right] \frac{\text{lb-force}}{\text{in.}^2} \times \frac{144 \text{ in.}^2}{\text{ft.}^2} \times \frac{\text{ft.}^3}{69.9 \text{ lb-mass}}$$

$$\times \frac{0.997}{0.970} = 10.8 \frac{\text{ft., lb-force}}{\text{lb-mass}} .$$

The average linear velocity of the suspension, v_s, was

$$\frac{79.4 \text{ lb-mass}}{60 \text{ sec.}} \times \frac{0.997 \text{ ft.}}{69.9 \times 0.970 \text{ lb-mass}} \times \frac{1}{0.00133 \text{ ft.}^2} = 14.6 \frac{\text{ft.}}{\text{sec.}} .$$

The Fanning friction factor, f, defined as

$$f = \frac{P_f g_c D}{2Lv^2} , \tag{8}$$

where D is the inside pipe diameter, L is the length of pipe in which the pressure drop occurs, g_c is a conversion factor in Newton's law of motion and the other terms are as described previously, is

$$f = \frac{10.8 \frac{\text{ft.,lb-force}}{\text{lb-mass}} \times 32.17 \frac{\text{ft.,lb-mass}}{\text{lb-force,sec.}^2} \times 0.0411 \text{ ft.}}{2 \times 7.95 \text{ ft.} \times (14.6 \frac{\text{ft.}}{\text{sec.}})^2}$$

$$= 0.00422 \ .$$

Assuming that the Fanning friction factor of a smooth copper tube is the same for a suspension as it is for a pure liquid, and neglecting the fact that isothermal conditions did not exist, estimates of the Reynolds number and the viscosity of the suspension are afforded. From a plot of the Fanning friction factor versus Reynolds number for a smooth copper pipe, such as is given in standard engineering texts [e.g., Walker, Lewis, McAdams and Gilliland (1937)] and reproduced in Figure 16, a Reynolds number, $Dv\rho/\mu$, of 136,000 is found which corresponds to a friction factor of 0.00422. The suspension viscosity indicated by the friction factor, when the density is 69.9 lb-mass/ft.^{3} x $\frac{0.970}{0.997}$ = 68.0 lb-mass/ft.^{3}, is therefore

$$\mu = \frac{Dv\rho}{136{,}000} = \frac{0.0411 \text{ ft.} \times 14.6 \times 3600 \text{ ft./hr.} \times 68.0 \text{ lb-mass/ft.}^3}{136{,}000}$$

$$\mu = 1.08 \ \frac{\text{lb-mass}}{\text{hr.,ft.}} \ .$$

Caldwell and Babbitt (1941), investigating clay and other suspensions flowing in round pipes, found that the pressure drop data for any

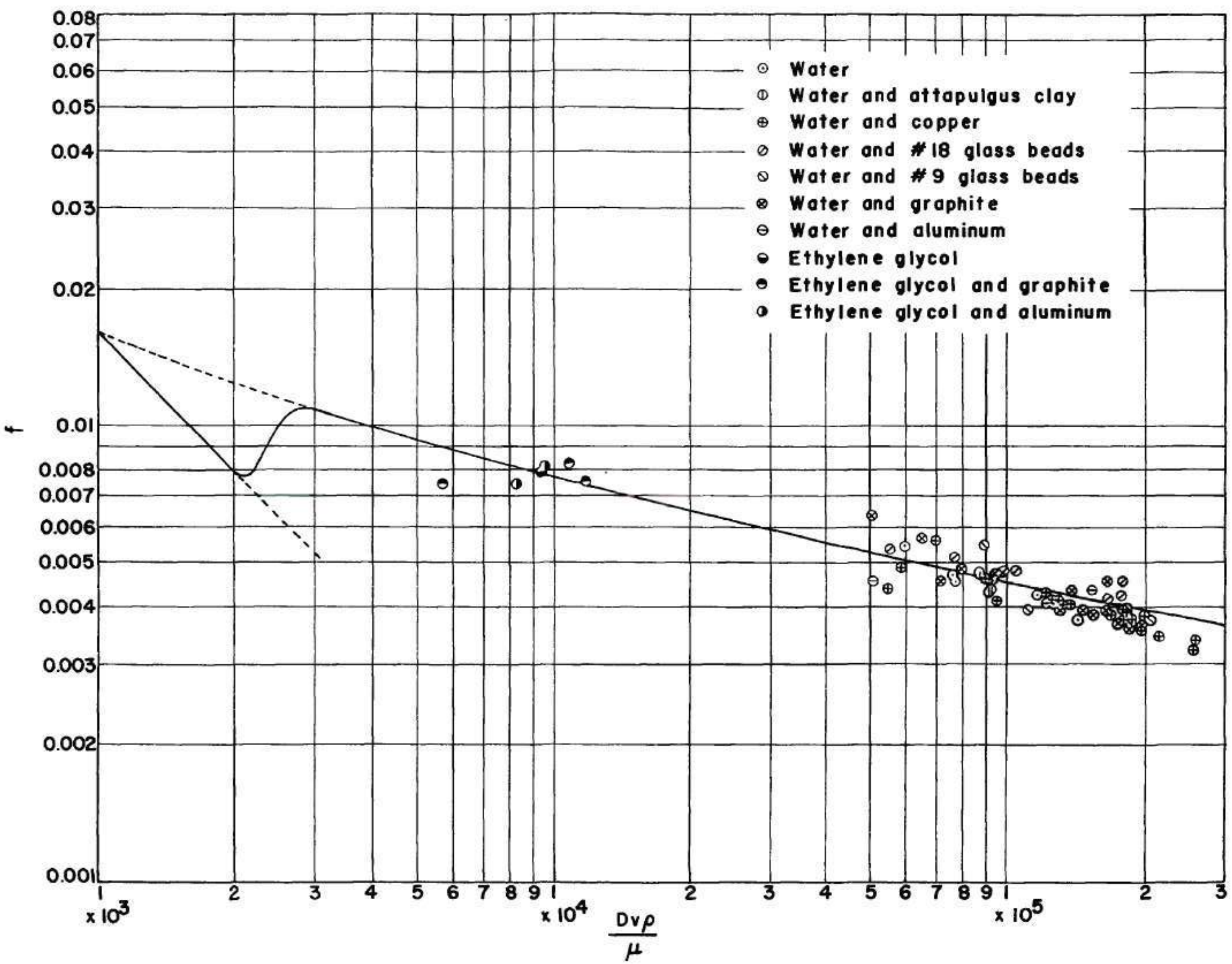


Figure 16. Fanning Friction Factor for the Nonisothermal Flow of Liquids and Liquid-Solid Suspensions.

one pipe could be represented on a friction factor plot such as Figure 16 by a single line if the Reynolds number were evaluated using the bulk mean velocity of the suspension, the density of the suspension and the viscosity of the pure liquid. Their investigation applied to isothermal conditions only. But to apply this finding to this investigation, the Reynolds number must be evaluated using mean temperature conditions. The viscosity of water at the bulk mean suspension temperature, as given by the data in the Appendix, is 0.833 lb-mass/hr.,ft.; therefore, the Reynolds number is

$$\frac{Dv\rho}{\mu} = \frac{0.0411 \text{ ft.} \times 14.6 \times 3600 \text{ ft./hr.} \times 68.0 \text{ lb-mass/ft.}^3}{0.833 \text{ lb-mass/hr.,ft.}}$$

$$= 176{,}000 \; .$$

This value may be found plotted versus the friction factor, 0.00422, in Figure 16.

While the data are not given, results for the isothermal flow of liquids in both the turbulent and laminar regions agreed closely with the curve of Figure 16. This was taken to mean that the pressure-measuring system functioned correctly.

As indicated previously, the ratio of suspension viscosities at bulk and wall temperatures raised to the 0.14 power will be required in the ultimate correlation. Since, as will be shown later, the viscosity of a suspension is conveniently represented by the viscosity of the continuous phase multiplied by a factor depending on the solid, its concentration, etc., the viscosity ratio can be evaluated from the liquid's properties only. Using the bulk suspension temperature, 181.4° F., the average inside pipe surface temperature, 228.3° F., and the viscosity data for water given in the Appendix, the ratio, raised to the power, is

$$\left(\frac{\mu}{\mu_w}\right)^{0.14} = \left(\frac{0.833}{0.620}\right)^{0.14} = (1.34)^{0.14} = 1.04 \ .$$

5. Calculated Results

With one exception, these results and corresponding results for the other runs are tabulated in Table III. Tabulated in the last two columns of Table III are data the significance of which is explained in the sections on thermal conductivity and viscosity; these data are included at this point so that Table III will contain the essential calculated data. Reynolds numbers are not presented at this point because their evaluation will be discussed in connection with the ultimate correlation, and another table is more appropriate for the presentation.

6. Analysis of Results

The discussion presented in this section must be limited to an evaluation of the consistency and general reliability of the data since a comprehensive appraisal appears at the conclusion of this report.

The so-called heat balance, the relation between the quantity of heat found to enter the system and that found to leave in a given unit of time, is the most obvious source of information. As may be seen from Table III, the discrepancy between the heat entering and leaving generally amounted to only a few per cent. Since a discrepancy of 10 per cent is not unusual in heat transfer work, it may be concluded that the measurements involved in obtaining the heat balance information are acceptable.

TABLE III

CALCULATED HEAT TRANSFER DATA

Run No.	Fluid Specific Heat (Btu / lb., °F.)	Heat into Exchange Section (Btu / min.)	Heat out of Exchange Section (Btu / min.)	Heat Balance Error (Based on Heat Entering) (Per Cent)	Logarithmic Mean Temperature Difference (°F.)	Heat Transfer Coefficient (Btu / hr., ft^2, °F.)	Fanning Friction Factor x 10^3	Viscosity from Fanning Friction Factor (lb. / hr., ft.)	Viscosity Ratio for liquid	Thermal Conductivity Ratio	Sedimentation Volume Fraction Solids
1	1.002	3440	3420	-0.60	55.8	3720	4.22	0.760	1.46	1.000	0
2	0.996	2970	2990	+0.7	53.4	3250	4.25	0.608	1.45	1.011	0.182
3	0.975	2980	2990	+0.03	52.3	3440	4.70	0.969	1.46	1.034	0.182
4	0.960	2890	2990	+3.5	48.8	3580	4.37	0.688	1.44	1.055	0.182
5	0.945	2570	2690	+4.5	49.2	3150	4.67	0.968	1.42	1.080	0.182
6	1.002	3730	3680	-1.3	51.7	4350	3.76	0.508	1.43	1.000	0
7	1.001	2760	2760	0	60.4	2760	4.59	0.827	1.54	1.000	0
8	1.001	3440	3420	-0.6	49.4	4200	4.06	1.28	1.44	1.000	0
9	0.995	3640	3640	0	48.2	4560	4.05	0.760	1.41	1.003	0.541
10	0.994	2370	2390	+0.9	60.7	2360	4.89	0.920	1.59	1.003	0.541
11	0.996	4240	4120	-2.8	56.3	4540	4.01	0.698	1.47	1.002	0.541
12	0.994	2570	2580	+0.4	58.8	2640	4.40	0.469	1.57	1.003	0.541
13	0.994	3340	3420	+2.4	59.2	3410	4.10	0.558	1.53	1.002	0.541
14	0.997	3570	3730	+4.3	44.0	4900	3.92	0.727	1.34	1.002	0.541
15	0.992	3610	3760	+4.1	41.1	5290	3.91	0.738	1.31	1.003	0.541
16	0.990	2770	2760	-0.4	54.8	3050	4.61	0.869	1.46	1.005	0.541
17	0.870	3920	3970	+1.3	46.4	5110	3.75	0.622	1.36	1.060	0.541
18	0.874	2760	2670	-3.3	41.7	4000	4.30	0.764	1.34	1.058	0.541
19	0.878	3210	3200	-0.3	38.5	5040	3.81	0.732	1.29	1.056	0.541
20	0.879	3960	4150	+4.6	34.5	6930	3.43	0.421	1.27	1.056	0.541
21	0.879	3690	3620	-1.9	39.4	5650	3.77	0.566	1.27	1.055	0.541
22	0.769	3770	3660	-2.9	32.4	7030	3.54	0.456	1.28	1.122	0.541
23	0.768	2880	2960	+2.8	38.7	4500	3.81	0.674	1.31	1.120	0.541
24	0.875	2280	2380	+4.4	40.0	3440	4.14	0.580	1.29	1.058	0.541
25	0.692	3610	3520	-2.5	35.4	6160	3.26	0.385	1.28	1.182	0.541
26	0.675	2520	2500	-0.8	52.5	2900	4.13	0.774	1.45	1.198	0.541
27	0.646	3620	3690	+1.9	41.8	5230	3.38	0.501	1.33	1.228	0.541
28	0.651	1460	1510	+3.4	40.2	2190	5.60	1.72	1.36	1.221	0.541
29	1.001	2270	2290	+0.9	54.4	2520	4.73	0.903	1.44	1.000	0
30	0.932	2430	2410	-0.8	49.4	2970	5.14	1.36	1.44	1.016	0.510
31	0.954	3470	3450	-0.6	43.0	4880	4.18	1.39	1.35	1.006	0.510
32	0.868	3470	3420	-1.4	42.4	4940	4.22	1.11	1.34	1.035	0.510
33	0.861	2700	2780	+3.0	51.5	3170	4.80	1.18	1.48	1.039	0.510
34	0.883	1750	1820	+4.0	54.7	1930	5.39	1.12	1.49	1.031	0.510
35	0.797	3250	3400	+4.6	54.3	3610	4.51	1.56	1.42	1.059	0.510
36	0.807	2680	2750	+2.6	56.0	2890	4.80	1.37	1.52	1.058	0.510
37	0.807	2940	2920	-0.7	42.9	4140	3.65	0.512	1.29	1.054	0.602
38	0.826	2330	2320	-0.4	50.1	2800	3.94	0.450	1.35	1.047	0.602
39	0.642	3040	2860	-5.9	51.0	3600	3.92	0.782	1.41	1.124	0.602

(Continued)

TABLE III (Concluded)

CALCULATED HEAT TRANSFER DATA

Run No.	Fluid Specific Heat (Btu / lb., °F.)	Heat into Exchange Section (Btu / min.)	Heat out of Exchange Section (Btu / min.)	Heat Balance Error (Based on Heat Entering) (Per Cent)	Logarithmic Mean Temperature Difference (°F.)	Heat Transfer Coefficient (Btu / hr., ft.^{2}, °F.)	Fanning Friction Factor x 10^3	Viscosity from Fanning Friction Factor (lb. / hr., ft.)	Viscosity Ratio for liquid	Thermal Conductivity Ratio	Sedimentation Volume Fraction Solids
40	0.720	1950	2060	+5.6	62.5	1880	5.44	1.81	1.49	1.085	0.602
41	0.628	1660	1740	+4.8	29.1	3450	3.73	0.600	1.21	1.124	0.602
42	1.002	2440	2450	+0.4	53.6	2750	4.76	0.996	1.46	1.000	0
43	0.973	2480	2510	+1.2	52.4	2860	4.60	1.13	1.46	1.040	0.264
44	0.933	1933	1890	-2.1	41.6	2800	4.71	0.792	1.33	1.140	0.264
45	0.934	2240	2200	-1.8	33.3	4060	3.94	0.672	1.25	1.141	0.264
46	0.931	--	3470	--	53.5	3920	3.91	0.657	1.43	1.141	0.264
47	0.899	3740	3700	-1.1	50.6	4460	3.82	0.604	1.41	1.220	0.264
48	0.898	2240	2340	+4.5	49.4	2740	4.79	1.10	1.36	1.220	0.264
49	0.895	1760	1900	+7.9	57.7	1840	6.38	2.40	1.55	1.225	0.264
50	0.875	4180	4010	-4.1	39.1	6460	3.59	0.476	1.32	1.281	0.264
51	0.876	2280	2370	+3.9	32.4	4250	3.65	0.695	1.17	1.262	0.264
52	0.852	2320	2420	+4.3	42.1	3330	3.94	0.564	1.33	1.316	0.264
53	0.851	1400	1510	+7.9	42.2	2000	5.63	1.63	1.35	1.349	0.264
54	0.810	2070	2210	+6.8	36.9	3390	4.38	1.00	1.28	1.478	0.264
55	0.812	3670	3480	-5.2	38.1	5820	--	--	1.30	1.471	0.264
56	0.776	2660	2690	+1.1	43.2	3720	3.69	0.512	1.34	1.589	0.264
57	0.770	1570	1480	-5.7	68.4	1390	4.56	0.753	1.62	1.600	0.264
58	0.754	2310	2420	+4.8	44.3	3150	4.55	1.49	1.36	1.715	0.264
59	0.750	1610	1520	-5.6	50.7	1920	4.82	0.836	1.65	1.678	0.264
60	1.002	1530	1560	+2.0	32.7	2800	5.42	1.25	1.28	1.000	0
61	0.970	2350	2370	+0.9	46.7	3040	4.09	0.647	1.38	1.045	0.121
62	0.970	3650	3490	-4.4	54.8	4020	4.39	1.13	1.44	1.045	0.121
63	0.956	1710	1780	+4.1	42.1	2450	4.53	0.498	1.41	1.063	0.121
64	0.957	3030	3010	-0.7	42.2	4330	--	--	1.34	1.062	0.121
65	0.921	2950	2910	-1.4	48.6	3660	--	--	1.40	1.119	0.121
66	0.920	2210	2210	0	55.6	2400	--	--	1.48	1.118	0.121
67	0.928	2580	2590	+0.4	46.2	3370	--	--	1.36	1.106	0.121
68	0.928	1720	1800	+4.6	55.8	1860	--	--	1.49	1.105	0.121
69	0.892	2300	2360	+2.6	47.5	2920	--	--	1.39	1.167	0.121
70	0.891	1440	1530	+6.2	59.3	1470	--	--	1.51	1.166	0.121
71	1.002	2960	2840	-4.1	51.6	3460	--	--	1.41	1.000	0
72	1.002	2520	2370	-5.9	51.9	2930	--	--	1.43	1.000	0
73	1.001	1360	1430	+5.1	70.1	1170	--	--	1.68	1.000	0
74	0.629	640	679	+6.1	71.6	540	7.94	11.6	2.82	1.000	0
75	0.586	789	846	+7.2	73.8	645	8.25	14.4	2.82	1.200	0.276
76	0.561	757	733	-3.2	72.2	633	7.56	10.1	2.59	1.333	0.276
77	0.552	441	494	+12.0	88.0	302	7.43	7.59	3.62	1.332	0.276
78	0.600	416	486	+16.8	66.5	378	7.43	6.98	2.61	1.119	0.089
79	0.574	345	395	+14.5	63.8	327	8.15	10.4	2.44	1.207	0.089

The measurements involved were those of the temperature of the entering and leaving fluid streams, the temperature of the leaving condensate stream, the conditions of temperature and pressure of the incoming steam, the flow rates of fluid and condensate, and the concentration of solids in the case of a suspension since this latter determined the suspension's heat capacity. The fact that good heat balances were obtained also indicates that the solid particles followed the temperature of the liquid closely.

A word of explanation is required as to why the condensate temperature was used in calculating the rate of heat input rather than the pressure under which it existed in the heat exchange section. Because of its being in contact with the cooler pipe, it was believed that the condensate might easily be undercooled, i.e., cooled below the temperature that its pressure would indicate if the condensate were assumed saturated. Using the condensate temperature involved no assumptions and introduced no comparable source of error. It will be noted, therefore, that the steam pressure in the exchange section, as indicated in Table II, is considered only approximate. Since the exact value was of little concern in this investigation, the exchange section pressure was used only as a guide for establishing and maintaining steady operating conditions. As described previously, the steam pressure was indicated by a Bourdon gauge on an exit line. The steam-side heat transfer coefficient, as indicated by the data, will be found to vary widely and to be considerably greater than would be predicted for film-type condensation. The steam was not filtered, and an inspection of the copper transfer pipe after several runs had been accomplished showed it to be covered with a thin, greasy film. The higher steam-side coefficient was probably due to some dropwise condensation.

Thermocouples were located at each end of the pipe only about 1/4 inch inside the heat transferring section. In Figure 15 it will be noted that the temperature indicated at this point at the fluid inlet end was considerably below the temperatures indicated for other sections of the pipe. The fact that a small zone near the inlet deviated from the characteristics of the other portions of the pipe is not surprising; the zone probably contributed to the undercooling of the condensate discussed above. The existence of the zone was otherwise neglected in the calculations.

It will also be noted that the pressure drop results presented in Figure 16, with the possible exception of those at the highest Reynolds numbers, indicate that the usual friction factor plot is applicable to the flow of suspensions when the viscosity of the pure liquid is used. This may be fortuitous, for Caldwell and Babbitt obtained somewhat different curves with different sized iron pipes, and agreement might not be as good with another apparatus. It is well to note that suspension pressure drop data for turbulent flow can be so represented for this case; it will be shown later that the heat transfer correlation requires a much higher viscosity than the viscosity of the liquid alone.

Water and aluminum react; the reaction proceeds so slowly under normal conditions that it is believed to have had a negligible effect on the heat transfer results.

B. Thermal Conductivity

1. Background

During the years preceding the Second World War, considerable attention was given in Germany to the problem of improving the thermal conductivity of electrical insulating materials. From his work on the subject, Meissner (1934a, 1934b, 1935) showed that the thermal conductivity of bituminous compounds, such as might be used to eliminate enclosed air spaces in certain electrical devices, could be increased several fold without appreciable detriment to the electrical properties of the compounds by the incorporation of divided solids such as quartz sand. Since this led to a saving of copper and a reduction in the size of certain electrical equipment, other investigators were attracted. Jackson (1942) in England, for example, conducted a rather thorough investigation of the properties of bituminous compounds and sand mixtures. Several other investigators studied the conductivity of these and other mixtures; one of these was the Russian, Tareef (1940), and it is his general relationship of the properties of the constituents and their concentrations that is of primary interest here.

Tareef's reasoning behind his selection is essentially as follows: The thermal field in a two-phase system is entirely analogous to the electrical field in a similar system. Since the equation describing the latter situation has been worked out by Maxwell and subsequent investigators, it may immediately be written for the thermal field. Therefore, the conductivity of a suspension may be written

$$k_s = k_L \left[\frac{2k_L + k_p - 2F\ (k_L - k_p)}{2k_L + k_p + F\ (k_L - k_p)}\right], \tag{9}$$

where the k's refer to the thermal conductivities of the suspension, the liquid and the solid particles instead of to electrical conductivities, and where F refers to the volume fraction of the particles.

Tareef checked the data of Meissner for bituminous compounds with quartz sand and for aniline gum with quartz sand using the above relationship and found good agreement. He attributed small discrepancies to faults in Meissner's technique. The writter checked the data of Jackson with equation 9 and found excellent agreement.

While this limited data indicated that equation 9 possessed considerable merit, it was deemed advisable to test it further and also to test it under more stringent conditions. Since water was of most interest as the liquid phase, some means was also sought by which water could be used and, at the same time, the settling out of the solid material prevented. This latter requirement was met by using a very dilute gel instead of pure water. The error introduced by so doing was small as will be shown later.

2. Apparatus

The construction and arrangement of the apparatus used to measure thermal conductivity is shown in Figures 17 and 18. It consisted of two chambers in which liquids could be maintained at constant but different temperatures and which were separated by two other chambers into which the suspension of unknown conductivity and a liquid of known

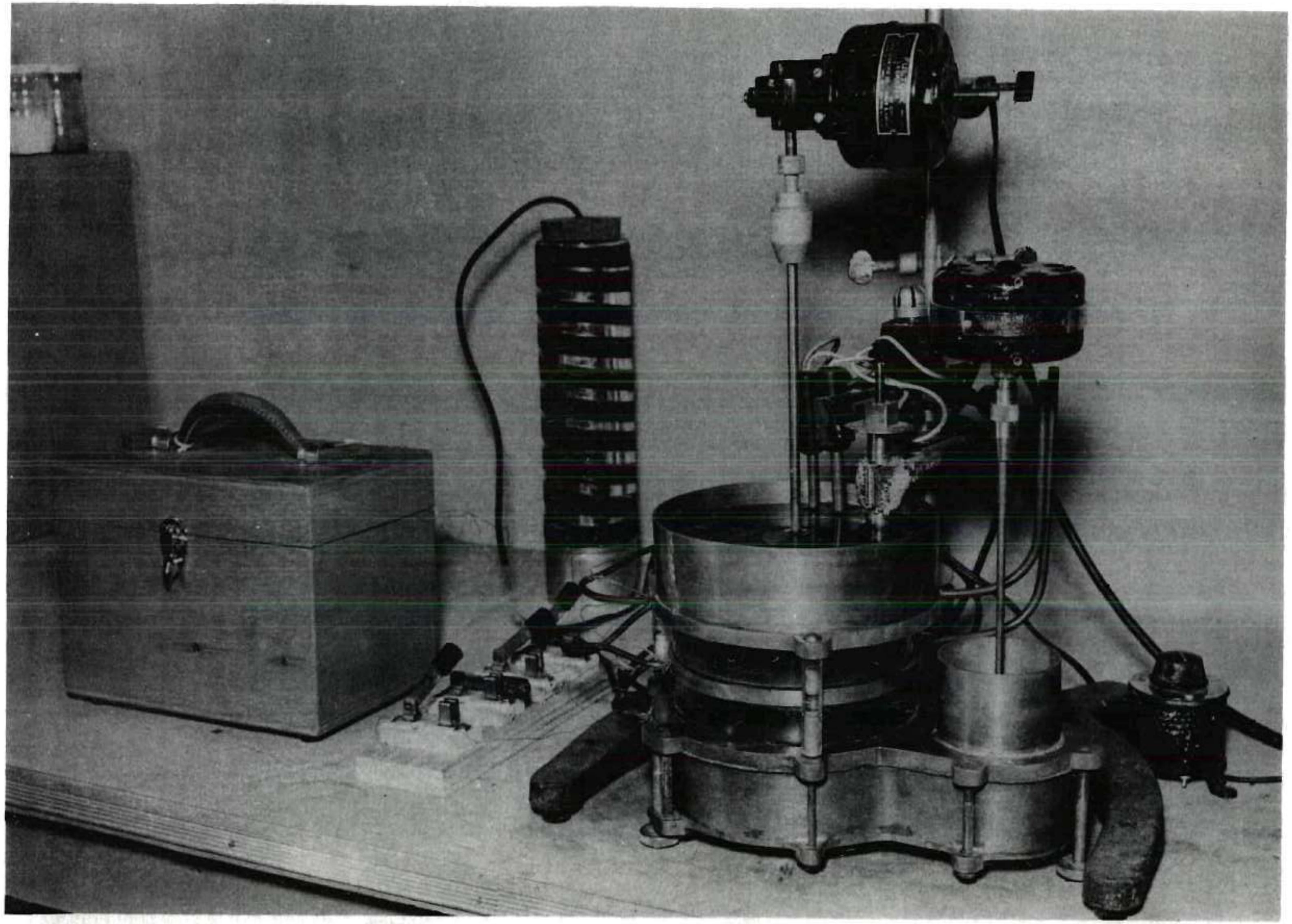


Figure 17. Thermal Conductivity Apparatus Without Insulation.

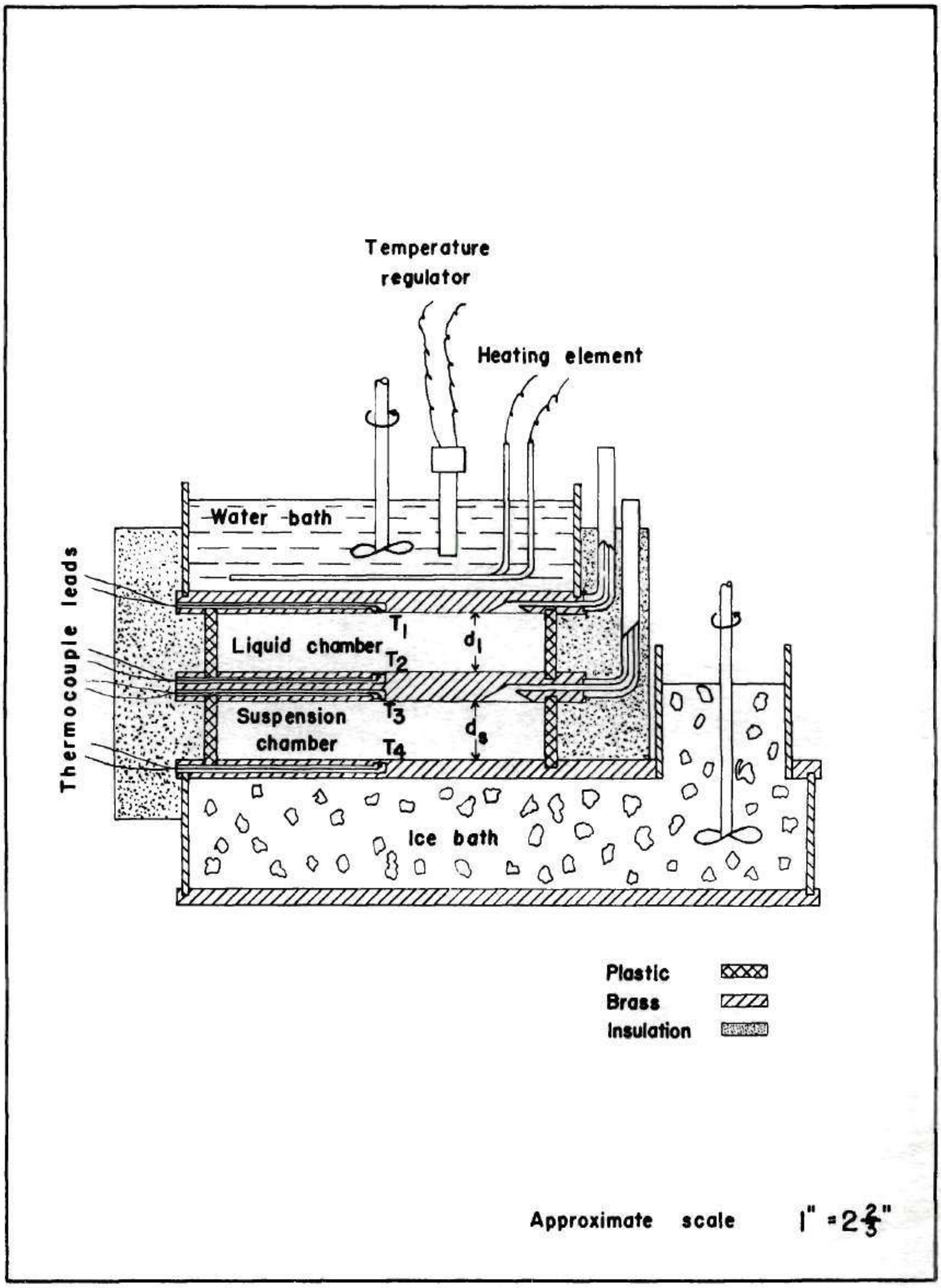


Figure 18. Schematic Diagram of Thermal Conductivity Apparatus.

conductivity could be placed. As may be seen from the figures, the pieces composing the walls of the middle conductivity chambers and other connecting parts were made of lucite, a material having a thermal conductivity somewhat less than that of water. When the chambers were externally insulated, therefore, heat flowed primarily through the contents of both chambers in a vertical direction. Inverted temperature conditions were employed to eliminate convection currents. The horizontal elements of each chamber, as well as all parts of the two constant-temperature chambers, were made of brass, a high-conductivity material. Each lucite piece was sealed to the brass plate immediately under it. Thermocouples were installed just under the surface of each plate, forming a part of the conductivity chambers. Installation was accomplished, as in the case of the thermocouples in the pipe wall described previously, by milling a groove in the plate surface, laying in the thermocouple insulated to the bead with a glass cloth sleeve, peening the bead into the plate, soldering a brass strip into the unfilled groove, and finally grinding and polishing the protrusions to the original plate surface level. As is shown below, the temperatures indicated by these thermocouples, the chamber dimensions and the conductivity of a reference substance gave sufficient information for the calculation of an unknown conductivity. The upper constant-temperature chamber contained a heating coil, a stirrer and a temperature-controlling device; the lower chamber, designed to make use of a water and ice mixture, required only a stirrer. The elevated lip around the opening into the lower chamber served to keep the liquid level always above the bottom of the lower conductivity chamber. The vertical tubes leading from each conductivity chamber facilitated the elimination of air bubbles if they occurred during the assembling of the apparatus and, in addition, served to keep the chamber completely filled at all times. The apparatus could be easily assembled and disassembled.

3. Experimental Procedure

Before every determination the brass surfaces of the apparatus through which heat was to be transferred were carefully polished with fine emery cloth. The suspension to be tested was prepared by dissolving two weight per cent agar, manufactured by the Difco Laboratories, Detroit, Michigan, in heated water and then adding the desired quantity of dried powder. While cooling, this mixture was stirred only enough to maintain a uniform suspension; violent stirring which would have incorporated air bubbles was avoided. As soon as the resulting gel became sufficiently rigid to prevent sedimentation, the material was carefully poured into the lower conductivity chamber of the apparatus, the plate forming the bottom of the chamber above was put in place, pure water was poured into the upper conductivity chamber, the upper constant-temperature chamber was put into place, and the entire device was fastened into a single unit with the lucite connectors. Insulation in the form of asbestos cloth was wrapped about the apparatus, the thermocouple leads were connected to appropriate switches, water and a water and ice mixture were placed in the upper and the lower constant-temperature chambers, respectively, the heater and stirrers were turned on, and the device was allowed to come to thermal equilibrium. Occasionally ice was added to the lower chamber. Establishing equilibrium required 2 to 4 hours. After equilibrium was attained, as indicated by temperature constancy, the temperatures were recorded and the vertical dimensions of the conductivity chambers were measured. These varied somewhat because of the use of different gaskets. It should be explained that the suspension was always placed in the lower, and consequently cooler, conductivity chamber. This was done in order that there would be little chance of melting the gel. It was found experimentally that temperatures only slightly exceeding room temperature were best for the heated bath; higher temperatures tended to cause bubble formation under the upper plate of the liquid conductivity chamber.

4. Data and Results

Examination of the schematic diagram of the equipment, Figure 18, shows that the heat flowing downward from the higher-temperature bath to the lower-temperature bath passed through both the liquid of known conductivity and through the suspension of unknown conductivity. When steady-state flow was reached, the thermal conductivity of the suspension was readily calculated. The heat transmitted through the liquid chamber in unit time, q_L, since the conductivity changes very slightly with the small temperature changes involved, is given by

$$q_L = -k_L A_L \frac{\Delta T_L}{\Delta x_L} \tag{10}$$

where k_L is the thermal conductivity of the liquid, A_L is the cross-sectional area of the chamber and $\Delta T_L/\Delta x_L$ is the temperature gradient in the liquid. Likewise, the heat transmitted through the suspension chamber may be written

$$q_s = -k_s A_s \frac{\Delta T_s}{\Delta x_s} . \tag{11}$$

Since there can be no build-up of heat in the system and since the cross-sectional areas of the two chambers were identical,

$$q_L = q_s ,$$

$$k_L \frac{\Delta T_L}{\Delta x_L} = k_s \frac{\Delta T_s}{\Delta x_s} ,$$

and it is immediately seen that

$$k_s = k_L \frac{(T_1 - T_2)\, d_s}{(T_3 - T_4)\, d_L} . \tag{12}$$

Experimental data obtained for a number of systems are given in Table IV; calculated results are given in Table V. The conductivity of water, the reference liquid, was obtained from the plot of conductivity data given in the Appendix; it was evaluated at a temperature equal to the average of the temperatures at the top and at the bottom of the liquid chamber. From this value and the other experimental data, the unknown conductivity was calculated by equation 12. The value of k_s thus obtained was considered to be the conductivity of the system under investigation at its average temperature. To compare the unknown system with water, the water conductivity was also evaluated at the average temperature of the suspension, and the results were expressed as a reduced conductivity, i.e., the ratio of the conductivity of the suspension or other system to that of

TABLE IV

EXPERIMENTAL THERMAL CONDUCTIVITY DATA

Fluid System	Weight of Solid (%)	Plate Surface Temperature T_1 (°F.)	T_2 (°F.)	T_3 (°F.)	T_4 (°F.)	Plate Spacing d_L (cm.)	d_s (cm.)
Water	0	98.4	64.0	64.0	32.9	3.05	2.53
Water and agar*	0	117.6	71.4	71.4	32.5	2.66	2.51
Water and agar*	0	100.5	64.5	64.5	32.8	2.99	2.50
Water, agar and powdered copper*	10.0	117.5	72.9	72.9	32.7	2.66	2.47
Water, agar and powdered copper*	50.0	116.6	64.4	64.4	32.7	2.66	2.47
Water, agar and powdered copper*	63.8	95.0	54.0	54.0	32.2	3.09	2.77
Water, agar and powdered copper*	73.3	96.0	52.1	52.1	32.3	3.03	2.98
Water, agar and powdered graphite*	9.9	116.6	69.4	69.4	32.7	2.66	2.48
Water, agar and powdered graphite*	40.2	150.2	68.6	68.6	32.3	2.72	2.51
Water, agar and No. 18 glass beads*	11.2	126.0	77.3	77.3	32.1	2.60	2.52
Water, agar and No. 18 glass beads*	38.2	100.2	61.9	61.9	32.4	3.01	2.53

*Agar present in an amount equal to two weight per cent of the water.

TABLE V

CALCULATED THERMAL CONDUCTIVITY RESULTS

Fluid System	Weight of Solids (%)	Measured Conductivity at Mean Temperature $\left(\frac{\text{Btu}}{\text{hr.,ft}^2 [^\circ\text{F. per ft.}]}\right)$	Reduced Conductivity*	Fluid System Conductivity at 175° F. $\left(\frac{\text{Btu}}{\text{hr.,ft}^2 [^\circ\text{F. per ft.}]}\right)$ Measured	Calculated**
Water	0	0.321	0.96	—	—
Water and agar***	0	0.398, 0.333	1.09****	0.429****	0.429
Water, agar and powdered copper***	10.0	0.367	1.09	0.429	0.435
Water, agar and powdered copper***	50.0	0.541	1.62	0.638	0.554
Water, agar and powdered copper***	63.8	0.583	1.76	0.693	0.694
Water, agar and powdered copper***	73.3	0.757	2.29	0.902	0.842
Water, agar and powdered graphite***	9.9	0.424	1.27	0.501	0.504
Water, agar and powdered graphite***	40.2	0.754	2.26	0.890	0.875
Water, agar and No. 18 glass beads***	11.2	0.375	1.11	0.436	0.437
Water, agar and No. 18 glass beads***	38.2	0.381	1.14	0.449	0.465

*The ratio of fluid system to water conductivity at identical temperatures.
**Assuming the measured reduced conductivity of water with agar to be correct.
***Agar present in an amount equal to two weight per cent of the water.
****Average value.

water. These results are shown in Table V. It may be noted that this ratio for water compared with water was 0.96. Ideally, it should have been 1.00. Similarly, when water was compared with water containing the small percentage of agar the ratio was 1.09. This indicates the agar increased the water's conductivity somewhat. Table V also shows a comparison of conductivity results calculated from the experimentally obtained reduced conductivity and from equation 9. With equation 9, the value for the conductivity of the liquid was taken as the experimentally determined value of the conductivity of water and agar; the conductivity of the solid was taken from the appropriate table and figures in the Appendix.

5. Sample Calculation

The data for the system containing 63.8 per cent copper by weight will be used for a sample calculation. By substituting the temperature and spacing data of Table IV and the conductivity of water as given in the Appendix into equation 12, a thermal conductivity for the system of

$$0.346 \frac{\text{Btu}}{\text{hr., ft.}^2 (^\circ\text{F. per ft.})} \times \frac{(95.0 - 54.0)\ 2.77}{(54.0 - 32.2)\ 3.09}$$

$$= 0.583 \frac{\text{Btu}}{\text{hr., ft.}^2 (^\circ\text{F. per ft.})}$$

is found. This is the conductivity of the system at its average temperature, 43.1° F. At this temperature pure water has a conductivity of 0.332 Btu/hr., ft. (°F. per ft.); hence, the reduced conductivity of the copper suspension containing the small percentage of agar is

$$\frac{0.583}{0.332} = 1.76 \quad .$$

This reduced conductivity, if employed to calculate the thermal conductivity of the suspension at another temperature, say 175° F., gives immediately

$$k_s = 1.76 \times 0.394 = 0.693 \frac{\text{Btu}}{\text{hr., ft}^2\text{, (°F. per ft.)}} ,$$

since the conductivity of pure water at this temperature is 0.394 Btu/hr., ft.^{2}(°F. per ft.). This value for the suspension's conductivity may be compared with that calculated from the generalized equation, equation 9. To make the results strictly comparable, the conductivity of the continuous phase, i.e., the water and agar, as indicated by experiment will be used instead of the conductivity of water alone. Therefore the conductivity of the continuous phase is

$$1.09 \times 0.394 = 0.429 \frac{\text{Btu}}{\text{hr., ft}^2\text{, (°F. per ft.)}} .$$

Since a suspension containing 63.8 weight per cent copper, the copper having a measured density of 513.8 lb-mass/ft.^{3}, is 17.2 volume per cent copper at 175° F., the conductivity predicted by equation 9 is

$$k_s = 0.429 \frac{(2 \times 0.429) + 218.8 - (2 \times 0.172)(0.429 - 218.8)}{(2 \times 0.429) + 218.8 + 0.172\ (0.429 - 218.8)}$$

$$= 0.694 \frac{\text{Btu}}{\text{hr., ft}^2\text{(°F. per ft.)}} .$$

For convenience in other calculations, the conductivities of the suspensions investigated in heat transfer have been expressed as a constant which, when multiplied by the conductivity of the pure liquid, gives the conductivity of the suspension. Returning to the calculation of run No. 32, which was halted on page 49, this factor may now be calculated. The conductivity of water at the mean temperature of the run, 181.4° F., is shown in the Appendix to be 0.397 Btu/hr.,ft.^{2}(°F. per ft.), the conductivity of the glass beads is given in Table IX as 0.67 Btu/hr.,ft.^{2}(°F. per ft.), and the volume fraction of solid material at the same temperature is also given in the Appendix as 0.0628. Therefore, substituting into equation 9, the conductivity of the suspension is found to be

$$k_s = 0.397 \frac{(2 \times 0.397) + 0.67 - (2 \times 0.0628)(0.397 - 0.67)}{(2 \times 0.397) + 0.67 + 0.0628\ (0.397 - 0.67)}$$

$$= 0.411 \frac{\text{Btu}}{\text{hr.,ft.}^2(^\circ\text{F. per ft.})} .$$

The suspension is then a better conductor than the liquid alone by a factor of

$$\frac{0.411}{0.397} = 1.035 .$$

This factor is given for all runs in Table III.

6. Conclusion

Examination of the results of the calculation above, the other results presented in Table V and the results of previous investigators indicates that equation 9 represents the conductivity of suspensions. In the case of the investigation presented here, the representation is well within the probable experimental error of the measurements. The equation will consequently be relied upon to give the suspension conductivity in the correlations of heat transfer data presented in another section.

The results of this investigation bear out two conclusions reached by Tareef, viz., (1) a very large increase in the thermal conductivity of a system cannot be obtained by selecting a filler, i.e., solid phase, of high thermal conductivity, and (2) to a first approximation the thermal conductivity of a system is independent of the degree of dispersion of the filler.

C. Viscosity

1. Background

As a first approximation, the viscosities of emulsions--liquid-liquid suspensions--are often assumed to be the weighted average of the viscosities of the individual components. If this situation prevailed in the case of liquid-solid suspensions their viscosities would rapidly approach infinity. Needless to explain, this is not the case, but the actual situation is none the less complex.

A fluid is said to be Newtonian when its viscosity is independent of the rate of shear. Liquids and dilute suspensions are approximately Newtonian in character. Concentrated suspensions, on the other hand, are most often non-Newtonian, i.e., their viscosities vary not only with temperature, as do the viscosities of all materials, but they vary with the rate of shear and in extreme cases vary with the duration of shear. This means, of course, that the apparent viscosity of a suspension may be a function of the pipe diameter and of the velocity of flow in the pipe if such is the system being employed. It means, moreover, that the viscosity of the suspension at various levels within the pipe may also differ, and that at certain concentrations and flow rates the suspension may cease its usual flow characteristics and begin slug (or plug) flow.

How some of these characteristics arise was considered by Bingham (1922) by analyzing the situation in the light of the liquid's motion and the consequent events that must occur to the particles. In discussing this approach, the settling of the solid particles because of gravity will be neglected, or, stated another way, the particles will be considered suspended in a liquid of their own density. If such a liquid is flowing without turbulence in a pipe, a velocity distribution is established across the pipe. Consequently, each stratum within the liquid is moving with a velocity different from that of its adjoining strata. If particles are present in the liquid, those in each stratum will be moving with different velocities. Moreover, the shearing within the strata will cause the particles to rotate. Particles in adjoining strata will approach each other because of their different velocities, but their surfaces will be moving in opposite directions, and the viscous resistance to this shearing action will dissipate their energy of rotation as heat as they approach.

Since particles which are large in comparison with molecular dimensions are being considered, the ordinary laws of friction are applicable. As long as the particles are in contact, they cannot rotate individually unless their torque exceeds a certain value; if the particles are spheres, the torque required is, of course, much less than if the particles are irregular. The value of the torque which would be necessary for the rotation of each type of particle depends upon the pressure normal to the surfaces at the point of contact. This pressure depends on the rate of shear as well as on the attraction or repulsion which may exist between the particles.

During the time of contact, if the particles cannot rotate individually, they begin to rotate as a whole. While they are rotating, liquid will flow around the particles and through the interstices between them. The result is that other particles collide with those already in contact, and the combined mass begins to rotate as a whole. When equilibrium is established, these agglomerates have a certain average size, depending upon the concentration, size and specific attraction of the particles and on the flow conditions.

There must be conditions of flow rate and concentration such that the agglomerates come in contact across the entire width of the passage; at this point, viscous flow of the suspension ceases and slug flow begins. If the rate of flow and, consequently, the shearing stresses, are increased above those prevailing when slug flow began, the average agglomerate size will be reduced and the flow will become viscous again. If the rate of flow is further increased so that turbulent conditions are obtained, the average agglomerate size will be drastically reduced because of the greatly increased shearing stresses; the apparent viscosity will likewise be drastically decreased.

While many expressions have been suggested by previous investigators with which it is possible to describe data for specific systems within limits, no generalized relationship has been developed by which the viscosity of a suspension at any set of conditions can be predicted with assurance. Only a few of the relationships that have been proposed—those that exhibit the general form of most—need be cited.

One of the best-known formulations is that of Einstein (1906, 1911). Assuming that the particles were spherical, large in comparison with the solvent molecules and uncharged, that there was no slip between the particles and the liquid (i.e., that the liquid adhered to their surfaces) and that turbulence was avoided, Einstein arrived at the relationship

$$\mu_S = \mu_L (1 + 2.5F) , \tag{13}$$

where μ_S is the apparent suspension viscosity, μ_L is the viscosity of the liquid at the same temperature, and F is the fraction of the total volume occupied by the particles.

The size of the solid particles is irrelevant as long as they are of a definite geometric shape. As Reiner (1949) points out, the viscous resistance of a material can be measured by the energy it dissipates per unit volume, and the additional energy dissipated per unit volume because of the presence of a rigid sphere, for example, can in no way depend upon the scale of the picture. For, if the linear scale is reduced, both the volumes of the sphere and of the liquid decrease as the third power of the reduction, and the volume concentration is not changed. But the radius is reduced as the first power and the surface area of the sphere is reduced as the second power. If, therefore, the scale is to have no influence, neither the radius nor the surface of the sphere can enter into the viscosity relationship. The above reasoning assumes the liquid to be homogeneous without limit. Since this is not so, the reasoning will break down when the scale is so reduced that the size of the particles approaches the liquid molecule size. The paragraph above does not imply that particle shape is without effect.

Equation 13 applies only at low concentrations. Eirich, Bunzl and Margaretha (1936) confirmed the equation for concentrations of spheres up to 5 or 10 per cent; McBain (1950), on the other hand, cites cases where values of the constant of 10 and 35 instead of 2.5 are required.

Hatschek (1913) proposed an equation of a form similar to that of Einstein with the constant equal to 4.5 instead of 2.5. He claimed the equation was applicable to a dispersed phase concentration of 40 volume per cent. Later, Hatschek (1920) proposed the relationship

$$\mu_s = \frac{\mu_L}{1 - F^{1/3}} \tag{14}$$

for emulsions and then justified it with results for a suspension of red-blood corpuscles. This is the equation used by Bonilla, et al. (1951) to give the viscosity of relatively dilute water-chalk suspensions. The equation, further discussed by Hatschek (1928), was derived for elastically deformable particles comprising more than one-half of the total suspension volume. It would seem to be a most inappropriate choice, but it actually describes many suspension data better than other equations.

More recently Norton, Johnson and Lawrence (1944) have developed the equation

$$\mu_s = \mu_L (1 - F) + aF + bF^c , \qquad (15)$$

where F is the volume fraction of solid material as before, and a, b and c are constants. According to the authors, the first term of the equation, $\mu_L (1 - F)$, expresses the part of the suspension's viscosity due to the medium; the second, aF, expresses the part caused by the rotation of the particles or groups of particles in the velocity gradient, and the last part, bF^c, expresses the part due to the interference of the particles or groups with each other. They found the equation to express the viscosity of clay suspensions to high concentrations. With three constants to be evaluated for each system, the equation is too unwieldy for use in an investigation of this type.

Vand (1948) developed theoretically the relationship

$$\mu_s = \mu_L (1 + 2.5F + 7.17F^2 + 16.2F^3) , \qquad (16)$$

in which the relative viscosity was expressed as a power series of the volume concentration. His results for glass spheres in a saturated solution of zinc iodide in a mixture of water and glycerol fitted the relationship reasonably well at low concentrations, but deviations above 25 volume per cent became quite apparent. The similarity of equation 16 to Einstein's, equation 13, is obvious. Guth and Simha (1936) have devised an expression in which the relative viscosity is also expressed as a power series of the volume concentration of the suspended material.

From the preceding discussion it is evident that the size distribution, shape and attractive forces of the particles should affect the viscous properties of suspensions but that they are not considered. While not taken directly into account, the relationship found by Bingham and Durham (1911) is dependent on these factors. Since the work of these investigators suggested the simple correlation to be presented below, their work will be dealt with in some detail.

Studying suspensions of infusorial earth, china clay and graphite suspended in water, as well as infusorial earth suspended in alcohol, Bingham and Durham found that, at any temperature, the fluidity, i.e., the reciprocal of viscosity, fell off essentially linearly with the solid concentration at low concentrations so that, if extrapolated, a fluidity of zero was indicated at a rather low concentration. The solid concentration at "zero fluidity" was found to be independent of temperature. Consequently, the suspension fluidity could immediately be expressed by

$$\phi_S = \phi_L \left(1 - \frac{F}{F_o}\right) , \tag{17}$$

where ϕ_S and ϕ_L are the fluidities of the suspension and of the pure liquid, respectively, F is the volume fraction of the solid material in the suspension as usual, and F_o is the volume fraction of solid material having the indicated zero fluidity. Expressed in terms of viscosities, equation 17 becomes

$$\mu_S = \mu_L \left(\frac{1}{1 - \frac{F}{F_o}}\right) . \tag{18}$$

Equation 18, if the value of F_o is known, expresses the viscosities of relatively dilute suspensions with considerable accuracy. Some of the suspensions used in this investigation had a solid concentration much greater than the indicated "zero fluidity" concentration, however.

2. Apparatus

Measuring the apparent viscosities of most suspensions is complicated by the fact that agitation must be provided to maintain the suspension. The problem, therefore, is to provide a suitable temperature-controlled measuring device having a stirring mechanism which does not greatly interfere with the measurement. DallaValle (1948) describes a viscosimeter developed at the U. S. Bureau of Standards which consisted essentially of a suspension container provided with one multipaddle stirrer and a capillary discharge tube. The apparatus devised for this investigation utilized these essentials but incorporated a number of refinements.

As may be seen in Figure 19, the main component of the apparatus was a commercial Saybolt viscosimeter manufactured by the Portable Products Corporation, C. J. Tagliabue Division, Brooklyn, New York. The instrument was equipped with a thermostatic device with which temperature could be controlled to 0.1° C. The instrument was altered in several ways; the alterations are best shown in the schematic drawing, Figure 20. First, the liquid chamber of the commercial apparatus, with its drain line and orifice, was replaced with another chamber and a capillary system. In this new component a capillary tube replaced the orifice, and the flow-controlling valve was built inside the fluid chamber. Sealing the flow

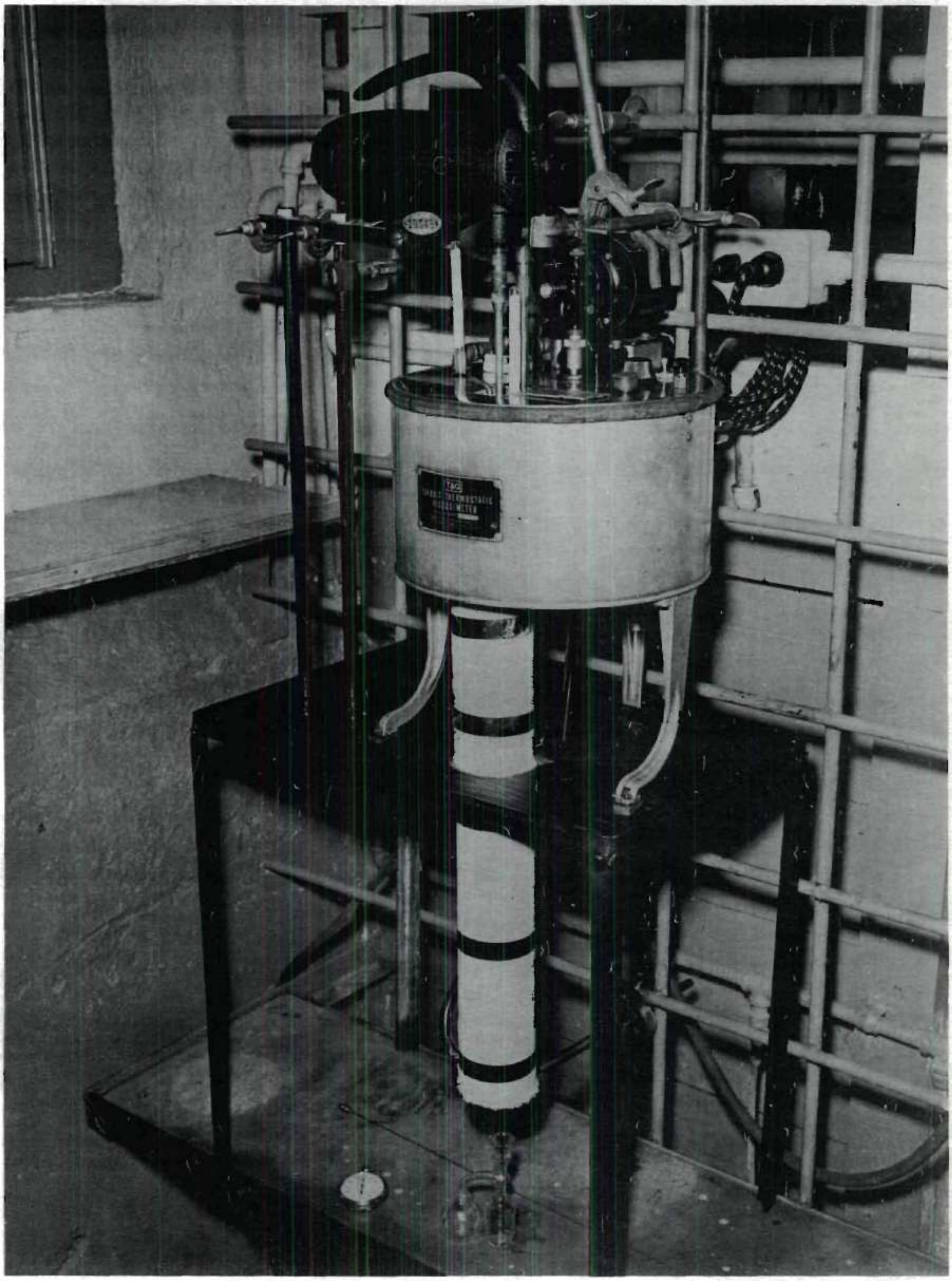


Figure 19. Suspension Viscosimeter Without Vacuum Attachments.

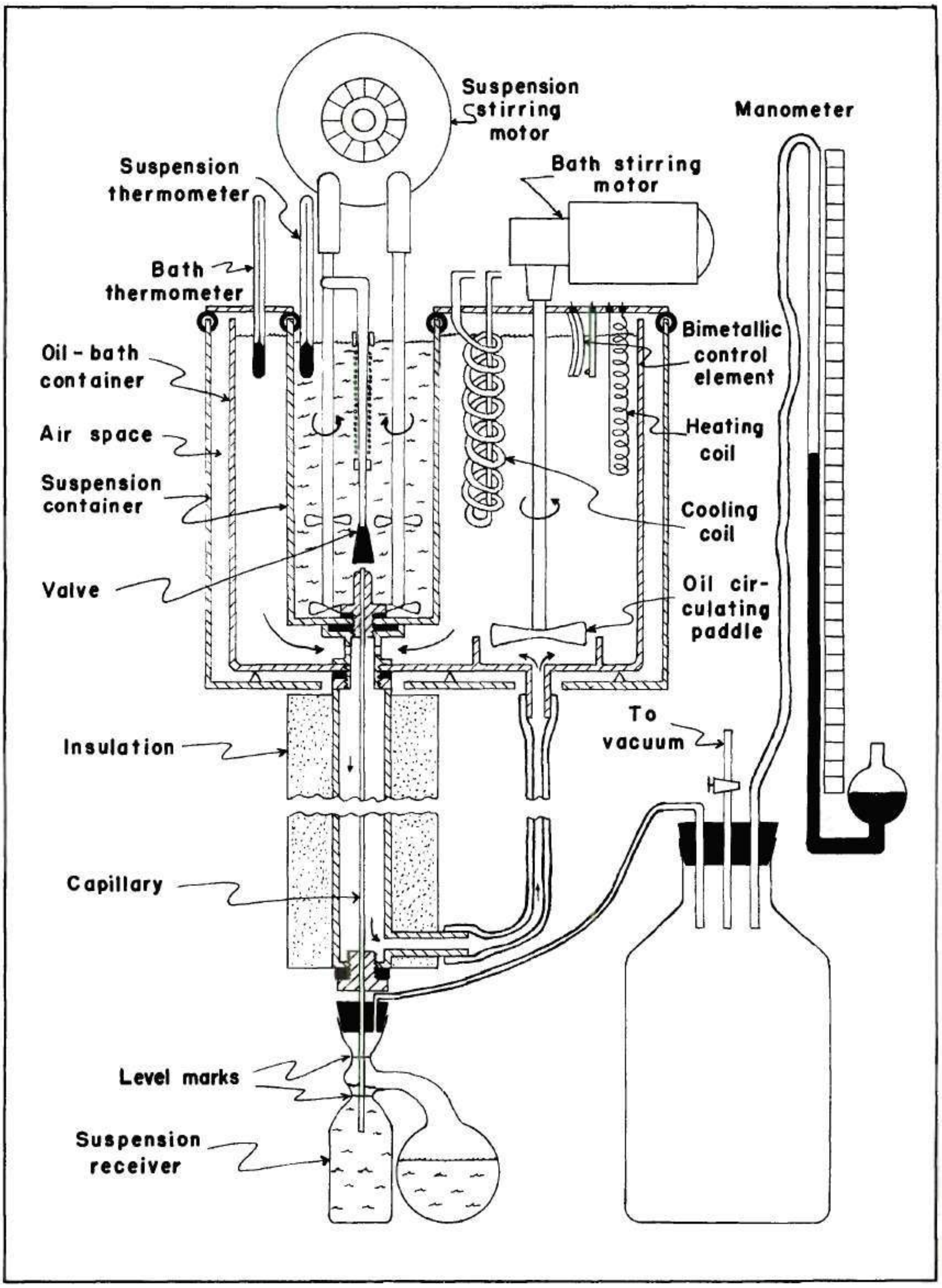


Figure 20. Schematic Diagram of Suspension Viscosimeter.

from above the capillary prevented the capillary from plugging with sediment. Second, the capillary made necessary an extension of the thermostated section. This was accomplished, as also shown in Figure 20, by surrounding the capillary with another tube through which the thermostated liquid, oil, was passed. The normal oil bath stirring system of the viscosimeter was modified to the extent of using a propellor of greater pitch to produce the necessary flow in the tube. Third, agitation of the suspension was accomplished by means of two oppositely rotating, multipaddle stirrers. The stirrers were located in relation to the orifice so that a relatively quiescent suspension entered the capillary. This, it is believed, decreased the so-called end effect, yet provided for an ideal suspension. Fourth, the suspension receiving bulb was specially designed to decrease the lower end effect and to provide a nearly constant pressure against which flow occurred during one run. As also shown in Figure 20, the receiving bulb was really two bulbs, the capillary extending into the left one. In making a run, timing of the flow through the capillary was not begun until this left bulb had filled to the lower level mark. Through nearly all of the run, therefore, the liquid level remained just that which was necessary to cause the liquid to run into the right and larger bulb. The run ended when the fluid level rose to the upper mark. Between the level marks a volume of 53.58 ml. was contained. Finally, arrangement was made whereby a barometer, a jar of approximately one and one-half cubic feet and an evacuating pump could be attached to the receiving bulb so that an additional pressure drop could be applied across the capillary.

Two capillary tubes, one having a diameter of 0.042 inch and one of 0.053 inch, were used in the investigation; both were 25.5 inches long. As may be seen from Figure 20, the lower end of the capillary tube extended into the fluid container so that the vertical distance between the levels in the two containers was 26.7 inches when a determination was started and, as a simple calculation will show, was 26.1 inches when the determination was completed. The fluid container of the viscosimeter was 2-5/8 inches in diameter.

3. Experimental Procedure

In making a viscosity determination, the viscosimeter chamber was filled to a fixed level with the liquid or suspension under investigation, and the liquid or suspension was allowed to come to temperature equilibrium. Then, while being stirred at a rate which was kept constant throughout the investigation, the seal on the capillary was released, allowing liquid or suspension to flow through the capillary into the receiving bulb. Timing with a stopwatch was begun when the fluid level reached the lower level mark on the receiving bulb and was stopped when the second level was reached.

If a checking determination was desired, the fluid was returned to the apparatus and the procedure was repeated. If a determination at a different flow rate was desired, the fluid was returned to the apparatus and the procedure was repeated for the new flow rate. Differing flow rates were obtained by partially evacuating the large jar which was connected to the receiving bulb. The pressure drop which then produced flow through the capillary was equal to the head of liquid or suspension plus the additional pressure drop produced by the evacuation of the jar. Because of the design of the receiving bulb and because of the large volume connected to it, the static pressure at the lower end of the capillary tube was considered as unchanging during all runs. Conditions were chosen so that streamlined flow was assured with few exceptions.

The liquid-liquid solutions used for calibrating the viscosimeter, as well as the liquid-solid suspensions to be tested, were generally compounded in the desired concentrations by mixing weighed amounts of each component. In some cases the suspension samples taken in the course of the heat transfer investigation were used. When a wetting agent was necessary, as in the case of the water and aluminum powder, it was added in the same proportions as used in the heat transfer investigation.

For reasons which will become more apparent later, the settled volume, i.e., the volume occupied by the solid component in a bed formed by gravity sedimentation from the liquid, was also determined for each system investigated. These data were obtained by mixing a weighed quantity of the solid material and the liquid in a graduated cylinder, allowing the suspension to settle under the influence of gravity for a long time, and determining the volume occupied by the sedimented bed. Some of the systems were allowed to stand for more than a month; however, a few days were found to be sufficient in every case. Again, the wetting agent, when used, was employed in the same proportion as it was in the suspension investigated for heat transfer properties.

The data taken were (1) the liquid or suspension temperature in the viscosimeter, (2) the concentration of solids in the suspension, (3) the diameter of the capillary employed, (4) the time required for 53.58 ml. of liquid or suspension to flow from the capillary, (5) the beginning and final pressure drop which produced the flow, and (6) the sedimented concentration of the solids. The first four items are recorded in Table VI, and the sixth is recorded in Table III; these values were obtained directly. The last item in Table VI is the mean pressure drop of the run; it was simply calculated from the experimental pressure drop data. How it was calculated and why recording the actual experimental data was deemed unnecessary is made evident in the following section.

4. Analysis and Method of Calculation

Since the level of the suspension in the viscosimeter decreased during a determination, the head or pressure causing the flow decreased. To find the mean head which caused the flow, the well-known equation of Poiseuille for viscous flow through a tube may be applied. The equation may be written in engineering units (A derivation is given in the Appendix.)

$$\Delta P = \frac{128\mu L dV}{\pi D^4 g_c dt}, \tag{19}$$

$$\text{or} \quad H = \frac{128\mu L dV}{\pi\rho D^4 g dt}, \tag{20}$$

where H is the head of fluid producing flow, V is the volume of fluid flowing in the time t, and the other symbols are as defined previously. If H_m is defined as a mean head, equation 20 may be written

$$H_m = \frac{128\mu LV}{\pi\rho g D^4 t}. \tag{21}$$

TABLE VI

EXPERIMENTAL VISCOSITY DATA

Fluid System	Fluid System Temperature (°F.)	Volume of Solids at Temperature (%)	Capillary Diameter (in.)	Time of Efflux of 53.58 ml. (min.)	Mean Pressure Drop Through Instrument (in. of suspension)
Water	86.2	0.0	0.042	2.140	26.4
Water	100.4	0.0	0.042	1.847	26.4
Water	122.0	0.0	0.042	1.556	26.4
Water	140.0	0.0	0.042	1.377	26.4
Water	86.0	0.0	0.053	0.919	26.4
Water and 15.62 wt. per cent glycerine	86.0	0.0	0.042	3.001	26.4
Water and 15.62 wt. per cent glycerine	86.0	0.0	0.053	1.257	26.4
Water and 31.53 wt. per cent glycerine	86.0	0.0	0.053	1.927	26.4
Water and attapulgus clay	96.8	0.868	0.042	2.043	26.4
Water and attapulgus clay	96.8	1.74	0.042	2.180	26.4
Water and attapulgus clay	96.8	2.46	0.042	2.270	26.4
Water and attapulgus clay	96.8	3.32	0.042	2.460	26.4
Water and attapulgus clay	96.8	4.24	0.042	2.667	26.4
Water and attapulgus clay	96.8	6.00	0.042	3.413	26.4
Water and attapulgus clay	96.8	7.59	0.042	4.190	26.4
Water and attapulgus clay	96.8	9.09	0.042	5.413	26.4
Water and attapulgus clay	96.8	10.9	0.042	8.303	26.4
Water and attapulgus clay	96.8	12.5	0.042	11.21	26.4
Water and attapulgus clay	96.8	2.22	0.053	0.970	26.4

(Continued)

TABLE VI (Continued)

EXPERIMENTAL VISCOSITY DATA

Fluid System	Fluid System Temperature (°F.)	Volume of Solids at Temperature (%)	Capillary Diameter (in.)	Time of Efflux of 53.58 ml. (min.)	Mean Pressure Drop Through Instrument (in. of suspension)
Water and attapulgus clay	96.8	3.03	0.053	1.027	26.4
Water and attapulgus clay	96.8	4.13	0.053	1.110	26.4
Water and attapulgus clay	96.8	7.25	0.053	1.520	26.4
Water and attapulgus clay	96.8	9.86	0.053	2.233	26.4
Water and attapulgus clay	96.8	10.3	0.053	2.733	26.4
Water and attapulgus clay	96.8	14.0	0.053	6.630	26.4
Water and attapulgus clay	96.8	5.50	0.053	1.310	26.4
Water and attapulgus clay	96.8	5.50	0.053	0.440	82.9
Water and attapulgus clay	96.8	5.50	0.053	0.413	102.3
Water and attapulgus clay	96.8	5.50	0.053	0.380	126.2
Water and attapulgus clay	96.8	5.50	0.053	0.363	137.6
Water and attapulgus clay	96.8	8.01	0.053	1.701	26.4
Water and attapulgus clay	96.8	10.3	0.053	0.680	89.2
Water and attapulgus clay	96.8	10.3	0.053	0.430	139.1
Water and attapulgus clay	96.8	10.3	0.053	0.337	184.4
Water and attapulgus clay	96.8	14.0	0.053	6.630	26.4
Water and attapulgus clay	96.8	14.0	0.053	1.747	80.2

(Continued)

TABLE VI (Continued)

EXPERIMENTAL VISCOSITY DATA

Fluid System	Fluid System Temperature (°F.)	Volume of Solids at Temperature (%)	Capillary Diameter (in.)	Time of Efflux of 53.58 ml. (min.)	Mean Pressure Drop Through Instrument (in. of suspension)
Water and attapulgus clay	96.8	14.0	0.053	1.230	129.0
Water and attapulgus clay	96.8	14.0	0.053	0.830	160.8
Water and attapulgus clay	96.8	14.0	0.053	0.607	212.6
Water and powdered copper	96.8	3.35	0.042	1.897	26.4
Water and powdered copper	96.8	4.60	0.053	0.833	26.4
Water and powdered copper	96.8	5.00	0.042	1.883	26.4
Water and powdered copper	96.8	6.70	0.042	1.860	26.4
Water and powdered copper	96.8	7.92	0.042	1.880	26.4
Water and powdered copper	96.8	8.08	0.053	0.837	26.4
Water and powdered copper	96.8	9.15	0.042	1.873	26.4
Water and powdered copper	96.8	10.8	0.042	1.873	26.4
Water and powdered copper	96.8	11.0	0.053	0.887	26.4
Water and powdered copper	96.8	12.5	0.042	1.980	26.4
Water and powdered copper	96.8	15.0	0.053	0.983	26.4
Water and powdered copper	96.8	17.0	0.042	2.313	26.4
Water and powdered copper	96.8	19.1	0.042	2.597	26.4
Water and powdered graphite	165.2	1.89	0.053	1.341	26.4

(Continued)

TABLE VI (Continued)

EXPERIMENTAL VISCOSITY DATA

Fluid System	Fluid System Temperature (°F.)	Volume of Solids at Temperature (%)	Capillary Diameter (in.)	Time of Efflux of 53.58 ml. (min.)	Mean Pressure Drop Through Instrument (in. of suspension)
Water and powdered graphite	165.2	2.00	0.053	1.360	26.4
Water and powdered graphite	165.2	4.74	0.053	1.638	26.4
Water and powdered graphite	165.2	6.98	0.053	2.134	26.4
Water and powdered graphite	165.2	9.01	0.053	2.670	26.4
Water and powdered graphite	165.2	10.8	0.053	3.935	26.4
Water and powdered graphite	165.2	13.8	0.053	6.985	26.4
Water and powdered graphite	165.2	16.7	0.053	15.64	26.4
Water and powdered graphite	179.6	1.89	0.053	1.209	26.4
Water and powdered graphite	179.6	2.00	0.053	1.226	26.4
Water and powdered graphite	179.6	4.74	0.053	1.493	26.4
Water and powdered graphite	179.6	6.98	0.053	1.909	26.4
Water and powdered graphite	179.6	9.01	0.053	2.377	26.4
Water and powdered graphite	179.6	10.8	0.053	3.434	26.4
Water and powdered graphite	179.6	13.8	0.053	6.230	26.4
Water and powdered graphite	179.6	16.7	0.053	14.15	26.4
Water and powdered graphite	194.0	1.89	0.053	1.132	26.4
Water and powdered graphite	194.0	2.00	0.053	1.196	26.4

(Continued)

TABLE VI (Continued)

EXPERIMENTAL VISCOSITY DATA

Fluid System	Fluid System Temperature (°F.)	Volume of Solids at Temperature (%)	Capillary Diameter (in.)	Time of Efflux of 53.58 ml. (min.)	Mean Pressure Drop Through Instrument (in. of suspension)
Water and powdered graphite	194.0	4.74	0.053	1.400	26.4
Water and powdered graphite	194.0	6.98	0.053	1.774	26.4
Water and powdered graphite	194.0	9.01	0.053	2.217	26.4
Water and powdered graphite	194.0	10.8	0.053	3.143	26.4
Water and powdered graphite	194.0	13.8	0.053	5.822	26.4
Water and powdered graphite	194.0	16.7	0.053	13.60	26.4
Water and powdered graphite	96.8	4.00	0.053	1.106	26.9
Water and powdered graphite	96.8	8.00	0.053	2.090	26.9
Water and powdered graphite	96.8	12.0	0.053	3.650	26.9
Water and powdered graphite	96.8	13.0	0.053	4.215	26.9
Water and powdered graphite	96.8	16.0	0.053	13.49	26.9
Water and powdered graphite	96.8	4.00	0.053	0.744	39.6
Water and powdered graphite	96.8	4.00	0.053	0.587	51.9
Water and powdered graphite	96.8	4.00	0.053	0.500	69.1
Water and powdered graphite	96.8	8.00	0.053	1.080	44.8
Water and powdered graphite	96.8	8.00	0.053	0.777	58.0

(Continued)

TABLE VI (Continued)

EXPERIMENTAL VISCOSITY DATA

Fluid System	Fluid System Temperature (°F.)	Volume of Solids at Temperature (%)	Capillary Diameter (in.)	Time of Efflux of 53.58 ml. (min.)	Mean Pressure Drop Through Instrument (in. of suspension)
Water and powdered graphite	96.8	8.00	0.053	0.670	66.8
Water and powdered graphite	96.8	8.00	0.053	0.553	80.8
Water and powdered graphite	96.8	8.00	0.053	0.423	100.3
Water and powdered graphite	96.8	12.0	0.053	0.660	83.5
Water and powdered graphite	96.8	12.0	0.053	1.289	47.1
Water and powdered graphite	96.8	12.0	0.053	0.973	63.3
Water and powdered graphite	96.8	12.0	0.053	0.657	81.4
Water and powdered graphite	96.8	13.0	0.053	1.830	43.7
Water and powdered graphite	96.8	13.0	0.053	1.100	62.9
Water and powdered graphite	96.8	13.0	0.053	0.697	90.8
Water and powdered graphite	96.8	16.0	0.053	2.030	79.9
Water and powdered graphite	96.8	16.0	0.053	1.519	96.3
Water and powdered graphite	96.8	16.0	0.053	1.112	117.1
Water and powdered graphite	96.8	16.0	0.053	0.670	174.9
Ethylene glycol and powdered graphite	122.0	7.41	0.042	20.98	26.4
Ethylene glycol and powdered graphite	122.0	13.0	0.042	38.13	26.4

(Continued)

TABLE VI (Continued)

EXPERIMENTAL VISCOSITY DATA

Fluid System	Fluid System Temperature (°F.)	Volume of Solids at Temperature (%)	Capillary Diameter (in.)	Time of Efflux of 53.58 ml. (min.)	Mean Pressure Drop Through Instrument (in. of suspension)
Ethylene glycol and powdered graphite	122.0	20.0	0.042	216.3	26.4
Water and No. 18 glass beads	96.8	15.0	0.053	1.023	26.4
Water and No. 18 glass beads	97.0	13.1	0.053	0.990	26.4
Water and No. 18 glass beads	96.8	5.00	0.053	0.917	26.4
Water and No. 18 glass beads	96.8	5.00	0.053	0.730	35.4
Water and No. 18 glass beads	96.8	5.00	0.053	0.680	41.6
Water and No. 18 glass beads	96.8	5.00	0.053	0.620	50.7
Water and No. 18 glass beads	96.8	5.00	0.053	1.429	17.5
Water and No. 18 glass beads	96.8	10.0	0.053	0.967	26.4
Water and No. 18 glass beads	96.8	10.0	0.053	0.954	26.9
Water and No. 18 glass beads	96.8	10.0	0.053	0.647	41.9
Water and No. 18 glass beads	96.8	10.0	0.053	0.770	34.1
Water and No. 18 glass beads	96.8	10.0	0.053	0.587	49.5
Water and No. 18 glass beads	96.8	10.0	0.053	0.557	59.2
Water and No. 18 glass beads	96.8	20.0	0.053	1.111	26.4
Water and No. 18 glass beads	96.8	20.0	0.053	0.889	33.4

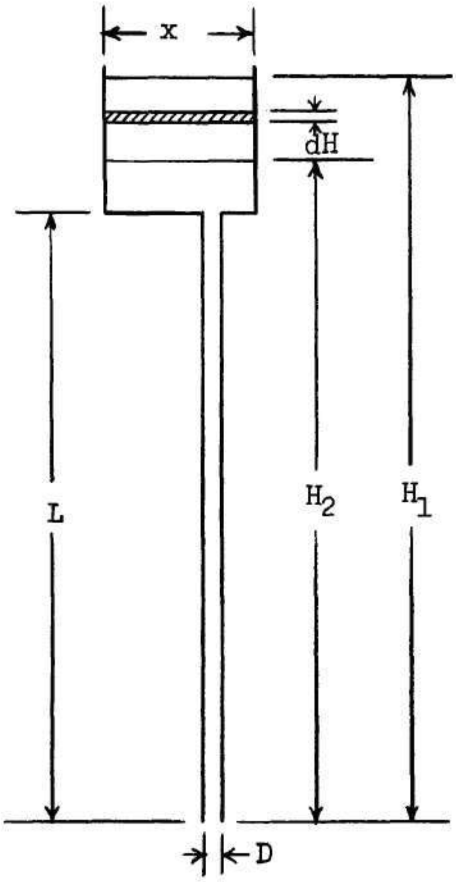
(Continued)

TABLE VI (Concluded)

EXPERIMENTAL VISCOSITY DATA

Fluid System	Fluid System Temperature (°F.)	Volume of Solids at Temperature (%)	Capillary Diameter (in.)	Time of Efflux of 53.58 ml. (min.)	Mean Pressure Drop Through Instrument (in. of suspension)
Water and No. 18 glass beads	96.8	20.0	0.053	0.770	39.8
Water and No. 18 glass beads	96.8	20.0	0.053	0.617	50.0
Water and No. 18 glass beads	96.8	20.0	0.053	0.540	58.2
Water and No. 18 glass beads	96.8	20.0	0.053	0.477	67.6
Water and No. 18 glass beads	96.8	30.0	0.053	1.386	26.4
Water and No. 18 glass beads	96.8	30.0	0.053	0.880	43.4
Water and No. 18 glass beads	96.8	30.0	0.053	0.573	70.9
Water and No. 18 glass beads	96.8	30.0	0.053	0.717	54.6
Water and No. 9 glass beads	96.8	9.40	0.053	1.210	26.4
Water and No. 9 glass beads	96.8	16.6	0.053	1.340	26.4
Water and No. 9 glass beads	96.8	21.6	0.053	1.490	26.4
Water and powdered aluminum	96.8	2.10	0.053	1.333	26.4
Water and powdered aluminum	96.8	3.25	0.053	1.770	26.4
Water and powdered aluminum	96.8	5.30	0.053	2.020	26.4
Ethylene glycol and powdered aluminum	96.8	2.10	0.042	35.77	26.4
Ethylene glycol and powdered aluminum	96.8	3.18	0.042	46.27	26.4

By referring to the accompanying sketch, it may be seen that



$$dV = -\frac{x^2\pi}{4}\ dH\ , \qquad (22)$$

and that, therefore, upon substituting equation 22 into equation 20, the relationship,

$$\int_{H_2}^{H_1} \frac{dH}{H} = -\int_{o}^{t} \frac{\pi\rho D^4 g dt}{32\mu x^2 \pi L}\ , \qquad (23)$$

is obtained, which, upon integration, becomes

$$\ln \frac{H_1}{H_2} = -\frac{\rho D^4 gt}{32\mu x^2 L}$$

$$= -\frac{\rho D^4 gt}{128\mu V} \cdot \frac{4V}{x^2 L}\ . \qquad (24)$$

Equation 24 may be rearranged to give

$$\frac{4V}{x^2\pi \ln \frac{H_1}{H_2}} = \frac{128\mu LV}{\pi\rho g D^4 t}\ . \qquad (25)$$

From equations 21 and 25 it may be seen that

$$H_m = \frac{4V}{x^2\pi \ln \frac{H_1}{H_2}}\ , \qquad (26)$$

while, from equation 22, if it is integrated between the limits of V = 0 and $V = V$ and $H = H_1$ and $H = H_2$, it is evident that

$$V = \frac{x^2\pi}{4} (H_1 - H_2) \; . \tag{27}$$

Combining equations 26 and 27, the result that

$$H_m = \frac{H_1 - H_2}{\ln(\frac{H_1}{H_2})} \tag{28}$$

is obtained. The values recorded in the last column of Table VI are this logarithmic mean head. Since the dimensions of the chamber, capillary lengths and the volume of fluid withdrawn were kept constant, the mean head remained constant and did not enter into the calculation of viscosity. (In five determinations the location of the receiving bulb was altered only very slightly, and the effect was neglected.) When an additional pressure drop was produced by creating a partial vacuum in the receiving bulb, this additional pressure, expressed in terms of height of fluid, was added to the logarithmic mean head which resulted from the column of fluid alone; these data were used in preparing shear diagrams as discussed below.

While the capillary viscosimeter is essentially a simple device in which a volume of fluid flows through a capillary tube of certain length and diameter under the influence of a given pressure in a certain time, every instrument of the type used here has a certain inherent defect for which correction must be made if accurate results are to be obtained. The liquid leaving as well as entering the viscosimeter capillary tube undergoes an acceleration. In addition, a certain degree of turbulence is introduced so that overcoming the friction within the capillary tube accounts for only a part of the work done by the driving force. The correction which must be applied is accordingly called the kinetic energy correction.

Poiseuille's equation, equation 19, may be written

$$\mu = \frac{\pi D^4 g_c t \Delta P}{128\ LV} . \tag{29}$$

Because it assumes that all energy is employed in overcoming the viscous resistance of the liquid, it is strictly applicable only to expressing conditions in a segment of a pipe where neither turbulence nor acceleration exists. Hagenbach (1860) first attempted to arrive at the kinetic energy correction, but Wilberforce (1891), detecting a slip in Hagenbach's reasoning, is generally given credit for arriving at the proper relationship. Bingham (1922) reports, however, that others had previously arrived at the same result. The corrected relationship is often written

$$\mu = \frac{\pi D^4 g_c t \Delta P}{128\ LV} - \frac{m \rho V}{8 \pi L t} , \tag{30}$$

where m is a constant that depends on the design of the viscosimeter.

Hall and Fuoss (1951) maintain that $m = 1$ corresponds to the case in which the flow is laminar or streamlined throughout the length of the capillary tube, but that in actual cases the amount of turbulence introduced at each end of the capillary tube makes the value of m unknown. They recommend, therefore, that equation 30 be accepted in form, because much experimental evidence indicates that it should be accepted, and that the actual analysis be made in a graphical manner. For a given instrument, using always the same head and permitting identical volumes of fluid to pass, equation 30 can be put in the form

$$\frac{\mu}{\rho} = at - \frac{b}{t} , \qquad (31)$$

where

$$a = \frac{\pi D^4 g_c \Delta P}{128\ LV} ,$$

and

$$b = \frac{mV}{8\pi L} .$$

Since ΔP and ρ, both variables, appear in the term for a, it might appear that a is not a constant. Actually, as the equation will be used here, the pressure drop and density are directly proportional, and a is a constant. A plot of $\mu t/\rho$ versus t^2 permits immediate evaluation of the constants a and b as the slope and intercept, respectively, of the resulting line. This procedure was employed in the calibration of the instrument using water and water-glycerine mixtures. The viscosities of these liquids as given by Lange (1949) were accepted as correct. The calibration curves and the resulting values for the constants a and b of equation 31 are shown in Figure 21.

The viscosities of the suspensions were calculated from the experimental data taken when the suspensions flowed under a pressure equivalent to their head only using the equations obtained by the calibration. The results are shown by Figures 22 through 27 with fluidity instead of viscosity plotted as a function of the concentration of solid material. Data of Bonilla, et al. (1951) and of Bonilla (1952), obtained for water-chalk suspensions using a similar instrument, are included in Figure 22.

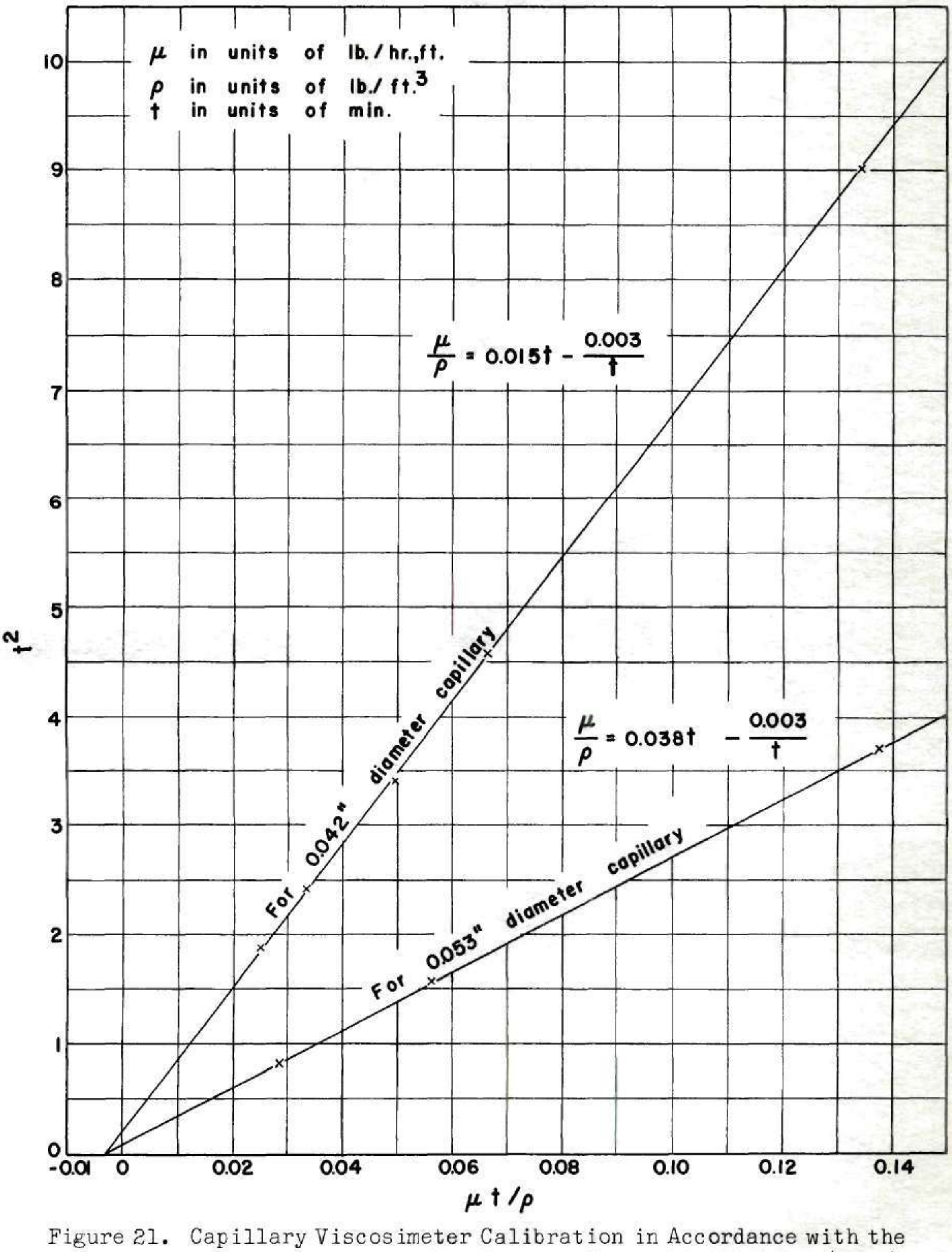


Figure 21. Capillary Viscosimeter Calibration in Accordance with the Empirical Relationship Suggested by Hall and Fuoss (1951). (The calibration is usable only when a mean liquid or suspension head of 26.4 inches exists.)

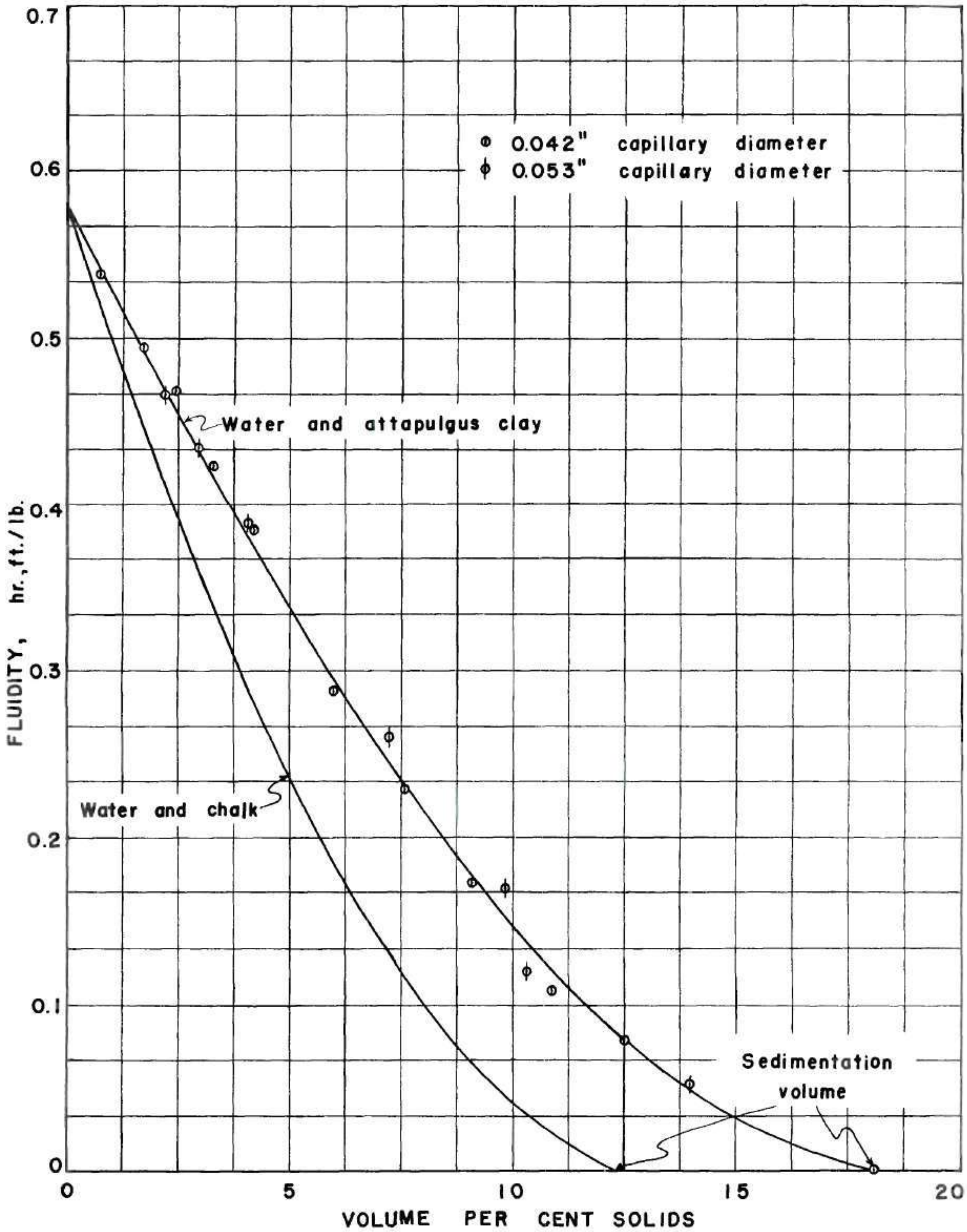


Figure 22. Fluidity of Water-Attapulgus Clay Suspensions at 96.8° F. and the Water-Chalk Suspension of Bonilla, et al. (1951) and of Bonilla (1952).

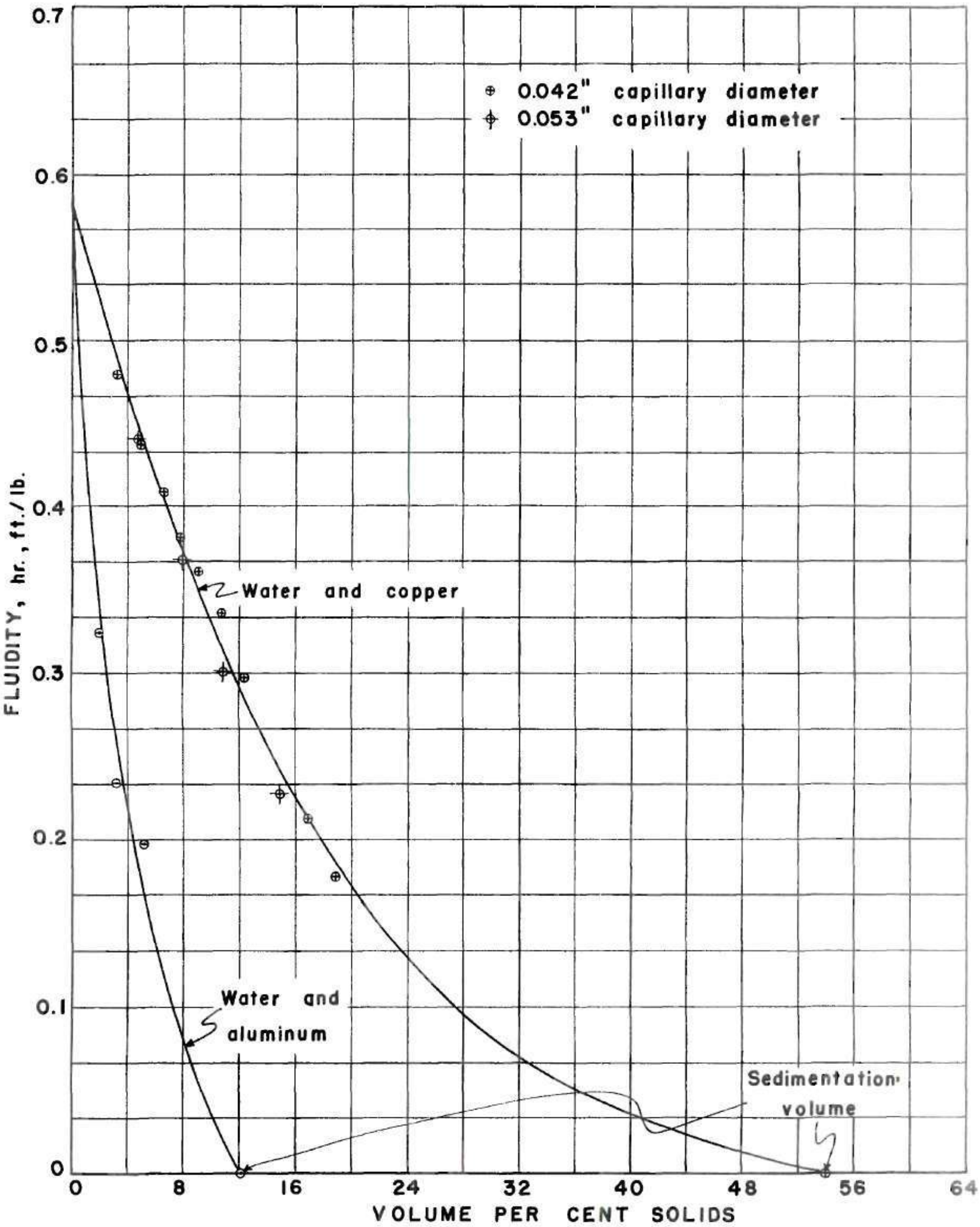


Figure 23. Fluidity of Water-Copper and Water-Aluminum Suspensions at 96.8° F.

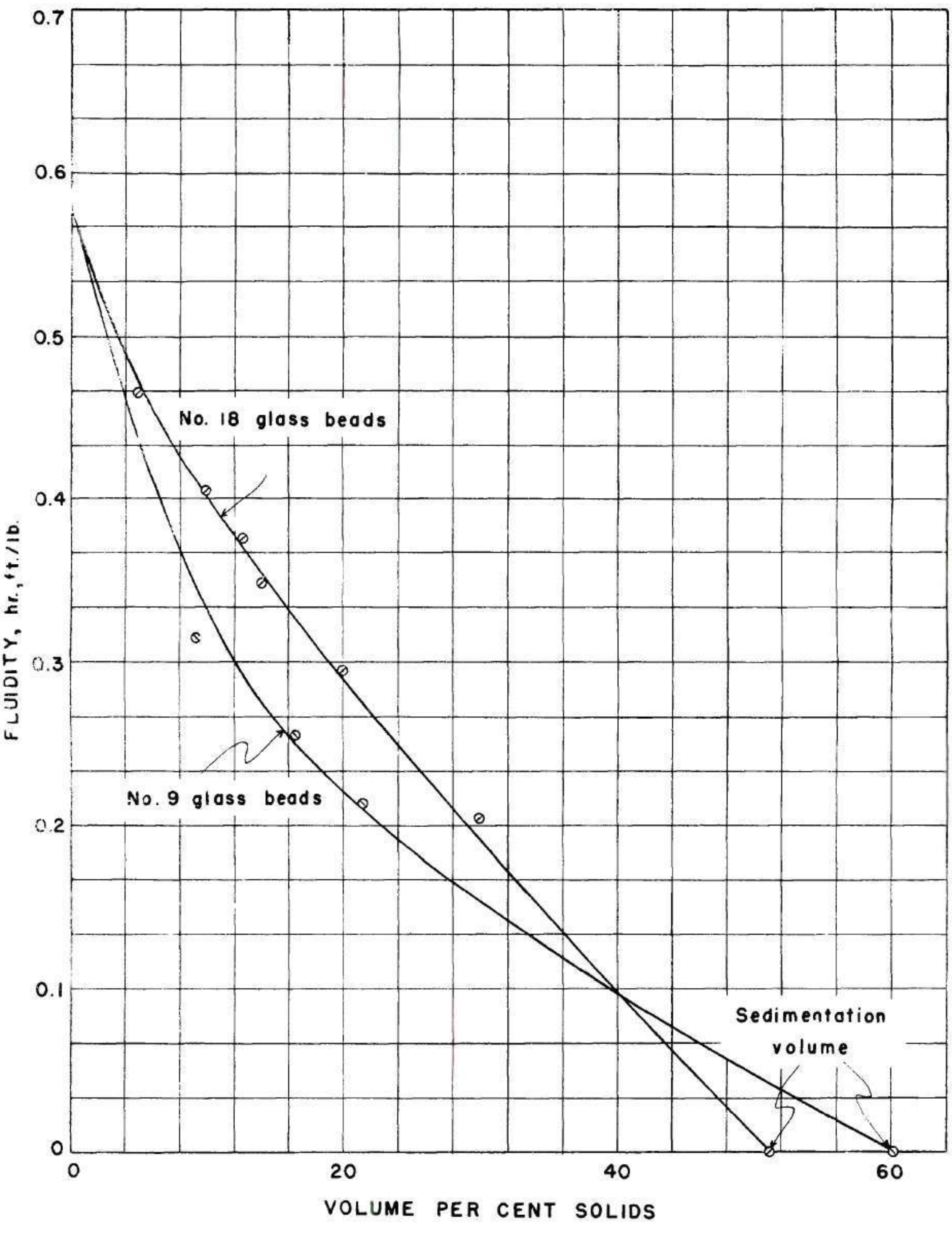


Figure 24. Fluidity of Water-Glass Bead Suspensions at 96.8° F.

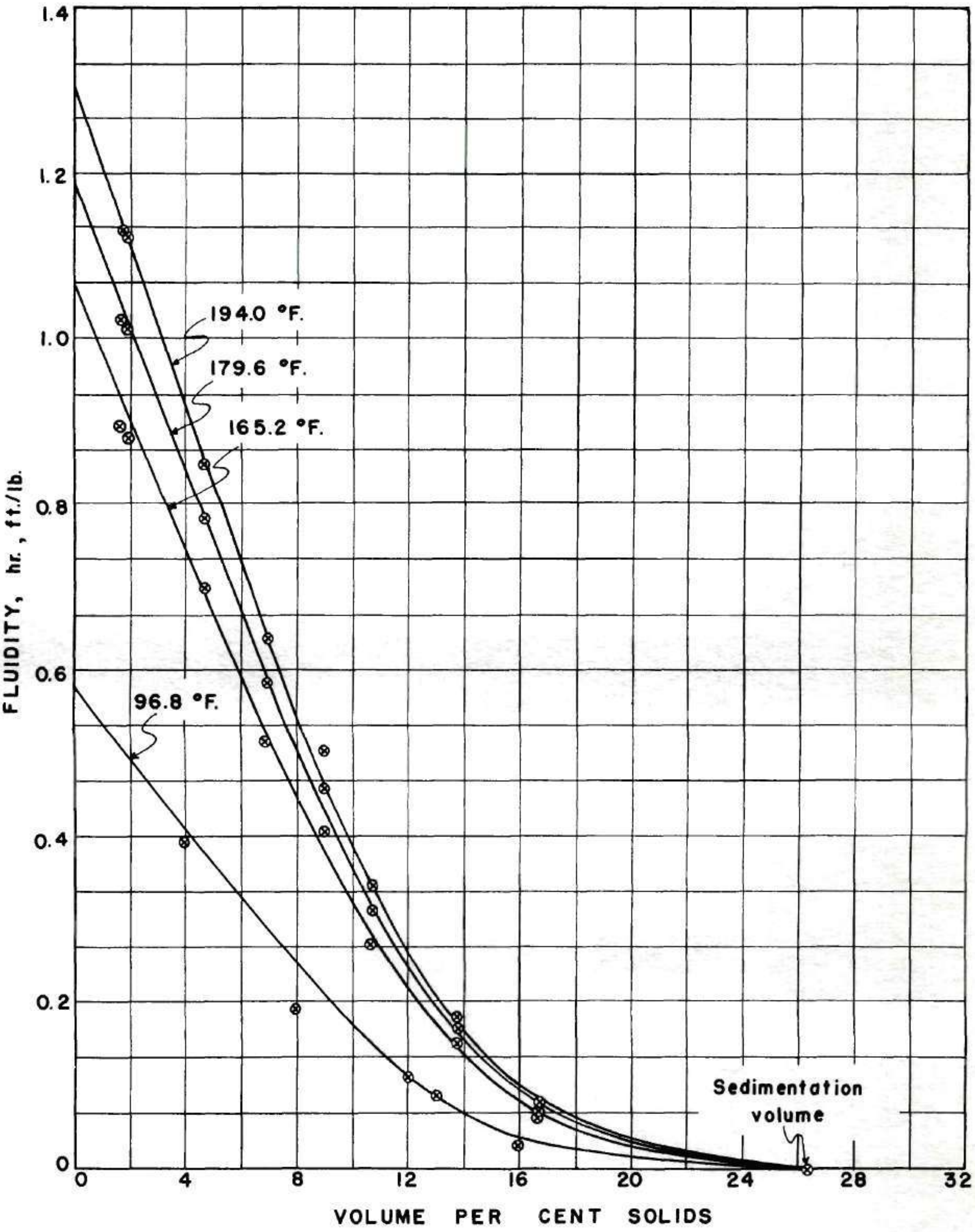


Figure 25. Fluidity of Water-Graphite Suspensions at Various Temperatures.

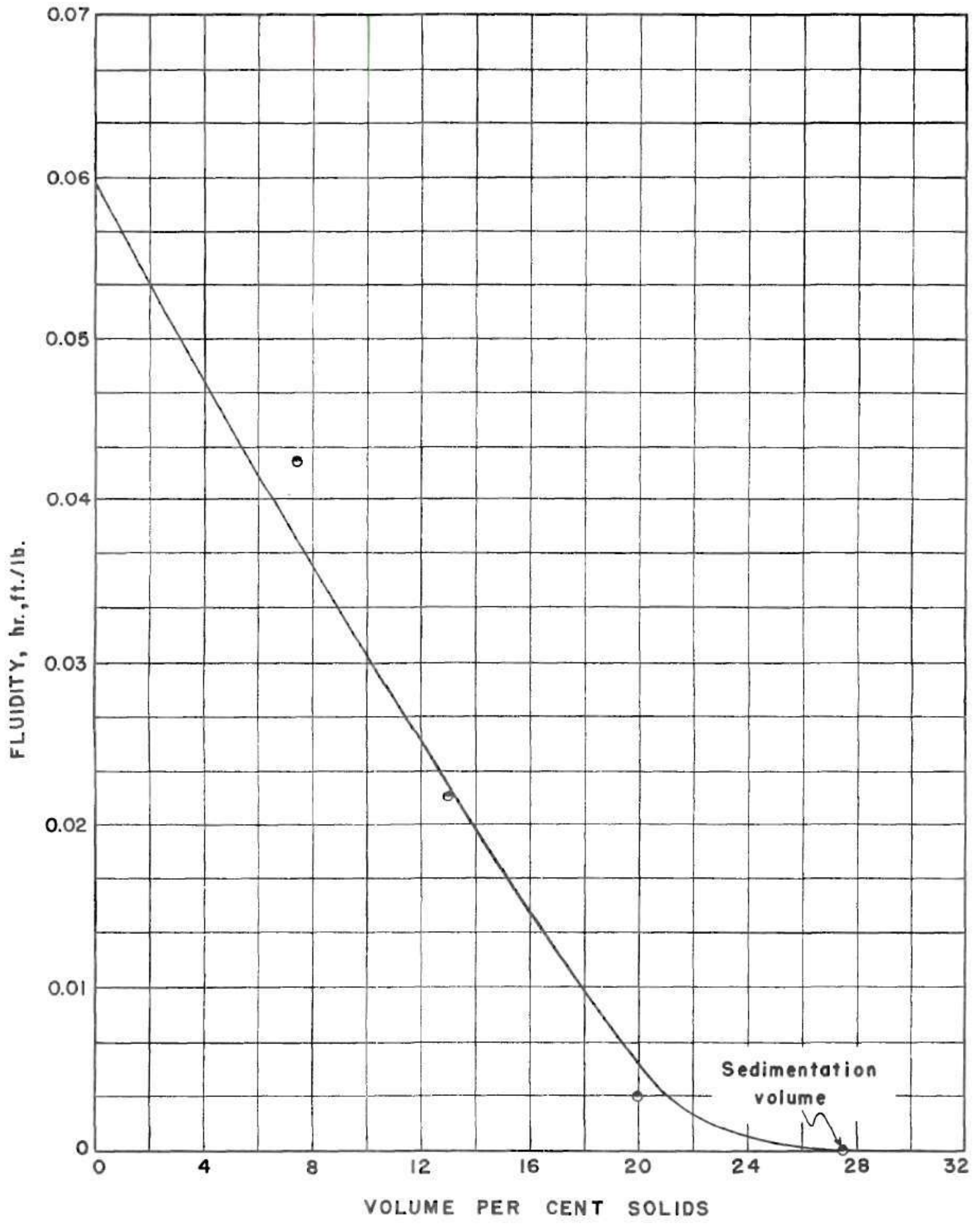


Figure 26. Fluidity of Ethylene Glycol-Graphite Suspensions at 122° F.

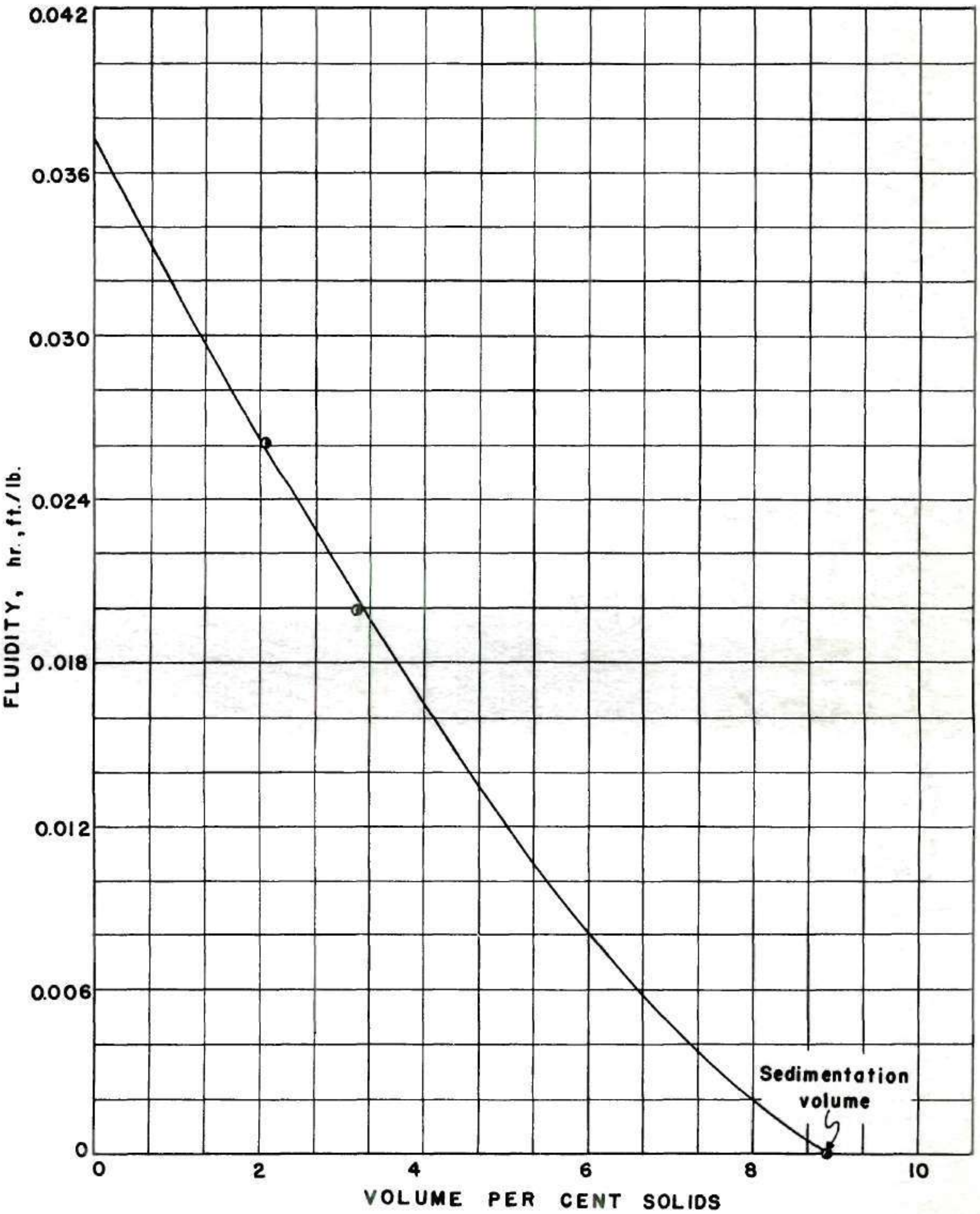


Figure 27. Fluidity of Ethylene Glycol-Aluminum Suspensions at 96.8° F.

Another procedure was followed for calculating and presenting the data obtained when a pressure drop greater than that due to the head alone was applied. The result is a better picture of the viscous behavior of a fluid although the analysis is not strictly correct. In the first place, the kinetic energy term discussed above is neglected. While this introduces an error of several per cent in the case of water, the error is negligible in the case of the more viscous suspensions. The second inaccuracy arises from the fact that a relationship applying to Newtonian fluids only is applied in the analysis of non-Newtonian ones. The analysis is widely used, however, for purposes of classifying fluids [See, for example, Alves (1949).], and the resulting plot is called a shear diagram.

If v is the velocity of flow, equation 29 can be put in the form

$$\mu = \frac{D^2 g_c \Delta P}{32\ Lv} = \frac{D^2 \rho\, Hg}{32\ Lv}\ . \tag{32}$$

Equation 32 can be written

$$\mu = \frac{g_c D}{8v} \cdot \frac{D \Delta P}{4L}\ , \tag{33}$$

and it may be seen that, on a plot of $8v/g_cD$ as ordinate and $D\Delta P/4L$ as abscissa, the viscosity is given by the inverse slope of the line through the origin and the particular conditions in question. Once the viscosity of a suspension is known, the Reynolds number that prevailed during the experimental run establishing the viscosity may be calculated also. Three shear diagrams, showing also several Reynolds number curves, are given in Figures 28, 29 and 30.

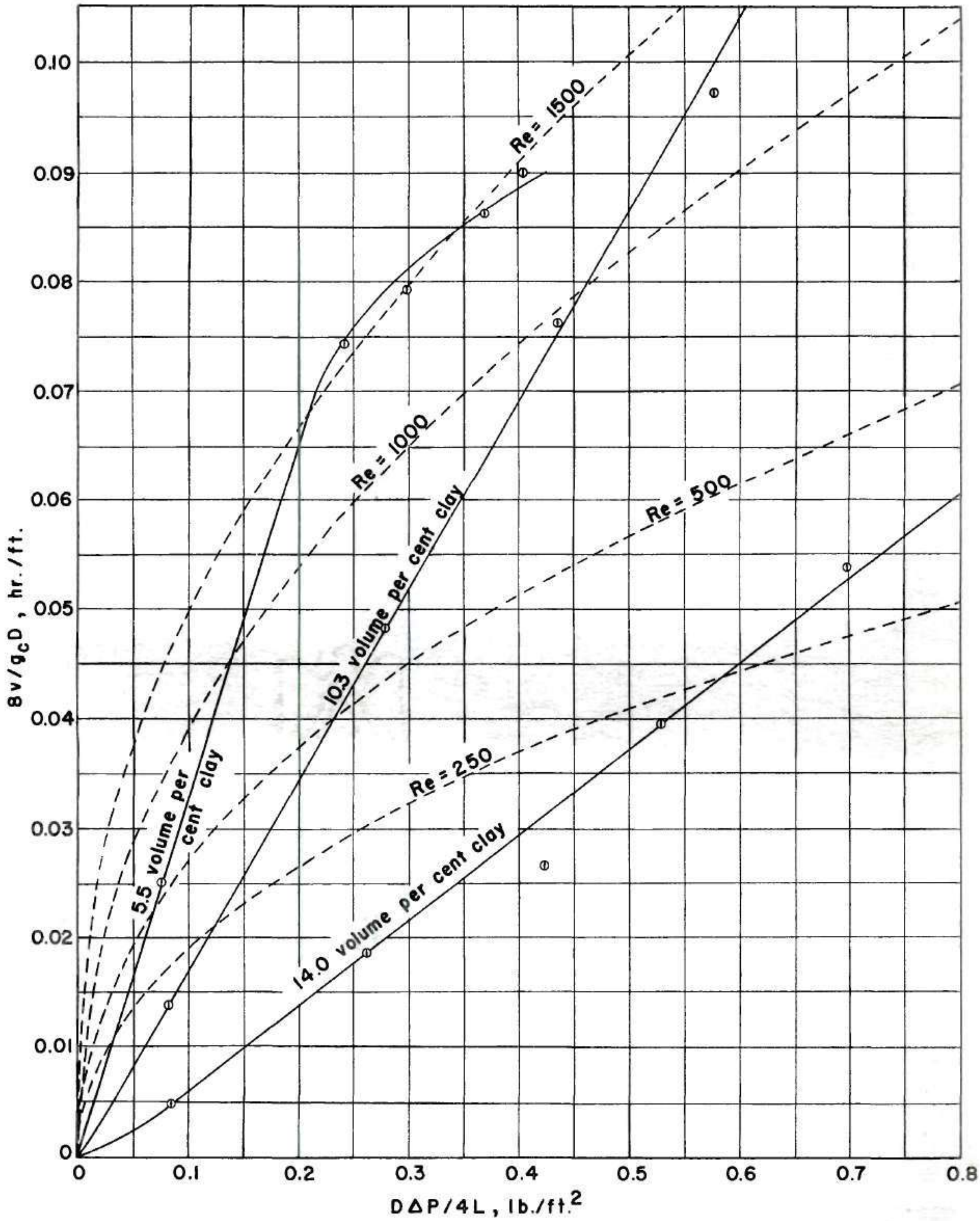


Figure 28. Shear Diagram for Water-Attapulgus Clay Suspensions at 96.8° F.

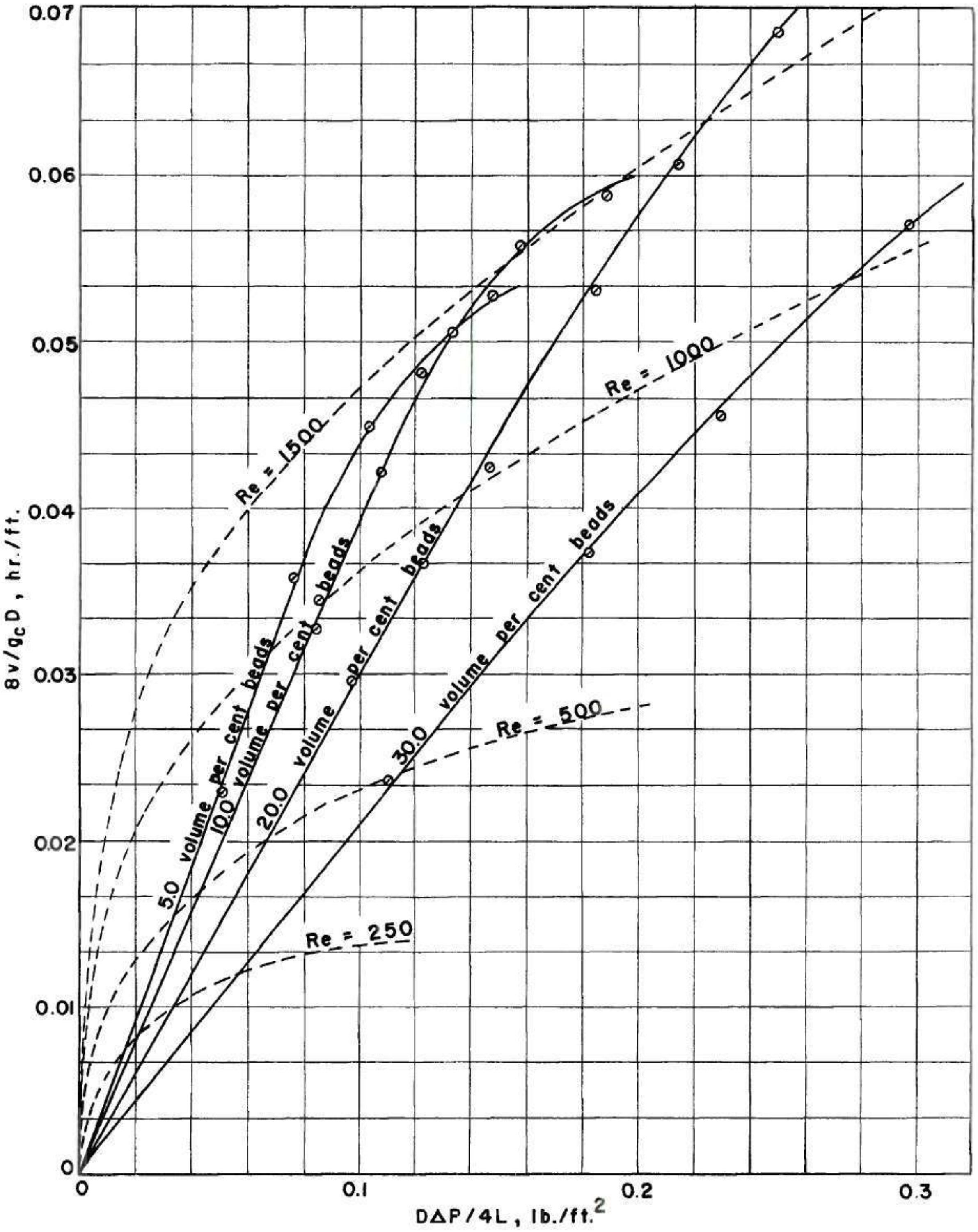


Figure 29. Shear Diagram for Water-Glass Bead (No. 18) Suspensions at 96.8° F.

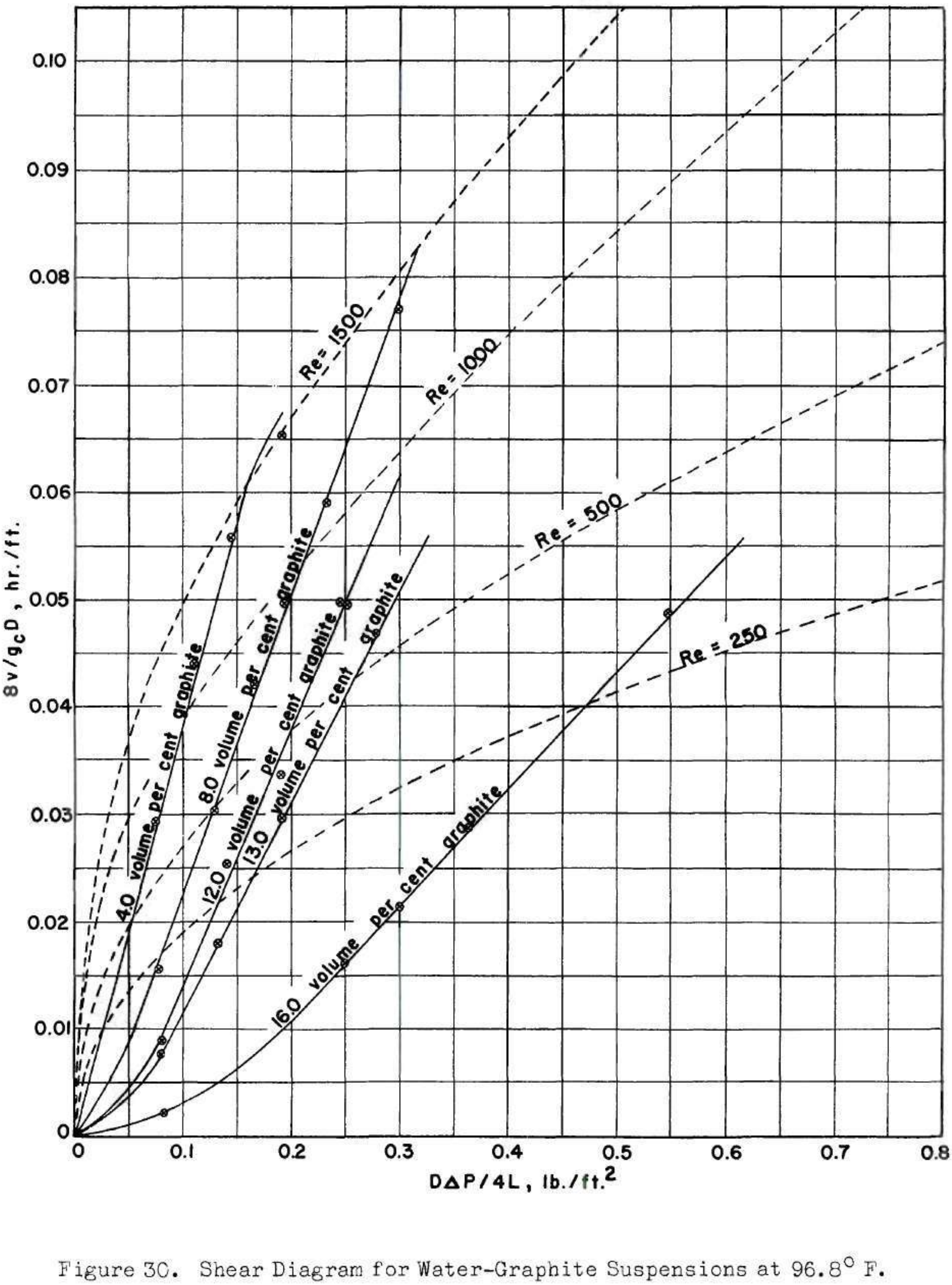


Figure 30. Shear Diagram for Water-Graphite Suspensions at 96.8° F.

A calculation of the quantities pertinent to a suspension of water and No. 18 glass beads is given below.

5. Sample Calculation

Referring to Table VI, it may be seen that a suspension of water and 20 volume per cent of No. 18 glass beads at a temperature of 96.8° F. flowed through the capillary tube having a 0.053-inch diameter when the pressure was equivalent to 26.4 inches of the suspension in 1.111 minutes. In Figure 21 the calibration for this tube is shown to have resulted in the equation

$$\frac{\mu}{\rho} = 0.038t - \frac{0.003}{t} , \tag{34}$$

in which viscosity is given in units of lb-mass/hr.,ft. when the density is expressed in lb-mass/ft.^{3} and time is expressed in minutes. The glass beads having an absolute density of 178.5 lb-mass/ft.^{3}, the suspension's density at 96.8° F. is readily calculated to be 85.3 lb-mass/ft.^{3} Therefore, substituting in equation 34,

$$\mu = (0.038 \times 1.111 \times 85.3) - \left(\frac{0.003 \times 85.3}{1.111}\right) = 3.37 \ \frac{\text{lb-mass}}{\text{hr.,ft.}}$$

or the fluidity, ϕ, is 0.297 hr.,ft./lb-mass. This point may be seen plotted in Figure 24.

A suspension of the same concentration, of identical materials and at the same temperature flowed through the same capillary under a pressure equivalent to 50.0 inches of the suspension in 0.617 minutes. Since 53.58 ml. of suspension flowed through in the time interval, the bulk mean velocity of flow was

$$v = \frac{53.58 \text{ ml.}}{0.617 \text{ x } 60 \text{ sec.}} \text{ x } \frac{\text{ft}^3}{28320 \text{ ml.}} \text{ x } \frac{4 \text{ x } 144}{(0.053)^2 \pi \text{ft}^2} = 3.34 \frac{\text{ft.}}{\text{sec.}}$$

and the first term of equation 33 is

$$\frac{g_c D}{8v} = \frac{32.17 \text{ ft.,lb-mass}}{8 \text{ lb-force sec}^2} \text{ x } \frac{0.053 \text{ ft.,sec.}}{12 \text{ x } 3.34 \text{ ft.}} \text{ x } \frac{3600 \text{ sec.}}{\text{hr.}}$$

$$= 19.16 \frac{\text{lb-mass,ft.}}{\text{lb-force,hr.}} .$$

The second term is more directly evaluated if differently expressed and then evaluated, thus, using density as given above and noting the capillary length of 25.5 inches,

$$\frac{D\Delta P}{4L} = \frac{D\rho Hg}{4Lg_c} = \frac{0.053 \text{ ft.}}{4 \text{ x } 12 \text{ x } 25.5 \text{ in.}} \text{ x } \frac{85.3 \text{ lb-mass}}{\text{ft}^3}$$

$$\text{x } 50 \text{ in. x } \frac{32.14 \text{ ft.,lb-force,sec}^2}{\text{sec}^2 \text{ } 32.17 \text{ ft.,lb-mass}} = 0.185 \frac{\text{lb-force}}{\text{ft}^2}$$

This point may be found plotted on Figure 29. (Note that the reciprocal of 19.16, or 0.0522, is plotted.) In Figure 29 a smooth curve has been drawn through this and the other experimental points for the same suspension. The viscosity of the suspension may then be found corresponding to any condition within the limits of the data. For example, when the bulk mean velocity of flow through the capillary tube of 0.053-inch diameter was 2.60 ft./sec., $8v/g_cD$ would be found to have a value of 0.0407 lb-force, hr./lb-mass,ft. The inverse slope of the line from this point on the 20 volume per cent curve to the origin, or more specifically, the viscosity of the suspension, is

$$\mu = \frac{0.138 \text{ lb-force,lb-mass,ft.}}{0.0407 \text{ ft}^2\text{,lb-force,hr.}} = 3.39 \frac{\text{lb-mass}}{\text{hr.,ft.}}$$

When the suspension's viscosity is established, the Reynolds number corresponding to the conditions of flow which produced that viscosity may be determined immediately. Thus,

$$\frac{Dv\rho}{\mu} = \frac{0.053 \text{ ft.}}{12} \times \frac{2.60 \times 3600 \text{ ft.,hr.,ft.}}{3.39 \text{ hr.,lb-mass}}$$

$$\times \frac{85.3 \text{ lb-mass}}{\text{ft}^3} = 1040 \ .$$

The broken line representing the locus of points of a Reynolds number of 1000 may be seen to pass through the point chosen for the example. That the calculation gave 1040 is due to the fact that, like all smooth curves based on experimental data, the curve for a Reynolds number of 1000 represents the best curve through several points.

The viscosity of the suspension obtained from the shear diagram, 3.39 lb-mass/hr.,ft., agrees exceptionally well with that calculated by equation 34, 3.37 lb-mass/hr.,ft.

The sample calculation for heat transfer run No. 32 may now be extended to the evaluation of viscosity. Because of the numerous relationships and correlations, results by several means will be collected while selection of the most applicable expression will be deferred to a later section. The suspension viscosity as indicated by the Fanning friction factor correlation has been calculated on page 46 with a result of

1.08 lb-mass/hr.,ft. The viscosity of the pure liquid at the bulk mean temperature, 181.4° F., as given by the data in the Appendix is 0.833 lb-mass/hr.,ft. Vand's relationship, equation 16, indicates therefore, a value of

$$\mu_s = 0.833\left[1 + (2.5 \times 0.0628) + (7.17 \times 0.0628^2) + (16.2 \times 0.0628^3)\right] = 0.990 \ \frac{\text{lb-mass}}{\text{hr.,ft.}}$$

Hatschek's relationship, equation 14, indicates a viscosity of

$$\mu_s = \frac{0.833}{1 - (0.0628)^{1/3}} = 1.38 \ \frac{\text{lb-mass}}{\text{hr.,ft.}} .$$

A value may also be obtained from the smoothed experimental data given in Figure 24. At the experimental conditions of measurement, 96.8° F., the suspension's indicated viscosity is

$$\frac{1}{0.450} = 2.22 \ \frac{\text{lb-mass}}{\text{hr.,ft.}} .$$

At this temperature, the pure liquid alone had a viscosity of 1.72 lb-mass/hr.,ft. At the temperature of the heat transfer run, the liquid had a viscosity of 0.833 lb-mass/hr.,ft. as shown above. Since, as will be shown below, a suspension's viscosity varies directly as the liquid's viscosity varies, the viscosity of the suspension as indicated by the experimental measurement is

$$2.22 \times \frac{0.833}{1.72} = 1.07 \ \frac{\text{lb-mass}}{\text{hr.,ft.}} .$$

The viscosity indicated by equation 35 (discussed later) is

$$\mu_s = \frac{0.833}{(1 - \frac{0.0628}{0.510})^{1.8}} = 1.05 \frac{\text{lb-mass}}{\text{hr.,ft.}}$$

where 0.510 is the fraction of the solid material found in a sedimented bed, a value recorded in Table III.

6. Results and Discussion of Results

Figures 21 through 30 present calibration curves, fluidity curves, and so-called shear diagrams as obtained from the experimental data. The volume fraction of each solid material in a sedimented bed is given in Table III.

As may be seen from Figure 21, the standardization data for the two capillary tubes plotted very straight lines having identical intercepts and different slopes. As may be further discovered, the values of the constant a are in the same ratio as are the capillary diameters raised to the 4th power while b is a true constant for the apparatus. That this should be so is shown by equation 31.

Figures 22 through 27 present the results obtained for the various suspensions when plotted according to the suggestion of Bingham and Durham, i.e., reciprocal of viscosity, the fluidity, against volume per cent of solid material. As further suggested by Bingham and Durham, it may be seen that the fluidity decreased in essentially a linear fashion with the solid concentration at relatively low solid concentrations so that, if this linear portion were extrapolated, a fluidity of zero would be indicated at rather low concentrations. Where the data are available, the concentration at which the fluidity would be zero appears to be independent of the temperature, moreover. As the concentration increases, however, the linear relation fails and the data indicate instead a fluidity of zero at a considerably higher concentration of the solid material. This higher concentration of zero fluidity is apparently also independent of temperature. By Figure 25, it may be seen that a given concentration of solid material changes the viscosity of the pure liquid the same relative amount regardless of temperature. This behavior is in agreement with the contention of Reiner (1949), who states that the change in the viscosity of a suspension with temperature is entirely due to the viscosity of the liquid.

Logically, a fluidity of zero (or an infinite viscosity) can occur only when the concentration is such that each individual particle is in contact on all sides with other particles; in other words, when it is in a bed. While it must be admitted that the measured fluidity-concentration data permit considerable leeway in locating the point of zero fluidity, smooth curves were obtained in every case when this point was taken as that representing conditions in a bed produced by gravity settling. This is the point plotted as a fluidity of zero on the figures. In Figure 22 the data obtained by Bonilla, et al. (1951), for a water-chalk suspension, using a gravity flow capillary viscosimeter of dimensions quite similar to the one used in this investigation, are also given. The data from which the sedimented bed concentration for the chalk system was calculated was kindly furnished by Prof. C. F. Bonilla (1952). While the chalk for this latter test was not from the sample used in the viscosity investigation, it was believed to be identical. It is of interest to note that Robinson (1949) used the assumption that the packed-sediment volume characterized the conditions for infinite viscosity, or zero fluidity, for a suspension of spheres.

The diagrams, Figures 28, 29 and 30, give information about three of the suspensions which the fluidity-concentration curves cannot give. As may be seen, all of these suspensions when dilute are essentially Newtonian, i.e., $8v/g_cD$, the rate of shear, is directly proportional to the shearing stress, $D\Delta P/4L$. At higher concentrations, water-glass bead suspensions become somewhat dilatant, the apparent viscosity increasing with increasing shear stress. This behavior is characteristic of quicksand, for example. On the other hand, both water-clay and water-graphite suspensions at the higher concentrations become less viscous with increasing shear stress. These latter suspensions may be classified either as Bingham plastics or as pseudoplastics. Absolute distinction cannot be made between the two types from the data; a Bingham plastic gives a positive intercept on the shear stress axis, while a pseudoplastic gives a curve coming to the origin. The suspensions appear to be pseudoplastics, however.

No points are found on the shear diagrams above a Reynolds number of about 1500. This is taken to mean that at this condition the suspensions began to flow turbulently. Alves (1949) shows that this behavior is typical of turbulent flow. A critical Reynolds number of 1500 is lower than usually found; it probably resulted from the fact that the stirring required for the suspension induced extra turbulence.

7. Correlation of Results

Examination of the fluidity results presented in Figures 22 through 27 shows that, while wide variation in the effect of the concentration of solid material is evident, all the curves are of the same general shape. In order to exploit this characteristic, a reduced plot of the fluidity data was made. The ordinate, instead of being fluidity alone, was now the fluidity of the suspension divided by the fluidity of the pure liquid, while the abscissa, instead of being simply the volume per cent solid material, was plotted as the volume fraction of solid material composing the suspension divided by the volume fraction of solid material which was found in a sedimented bed. The resulting plot is shown in Figure 31.

The broken line passing through the other curves on the figure and conforming to the empirical relationship

$$\mu_s = \frac{\mu_L}{\left(1 - \frac{F}{F_b}\right)^{1.8}}, \tag{35}$$

represents all of the data within approximately ± 15 per cent at two-tenths of the concentration of a sedimented bed, within ± 30 per cent at four-tenths, and within ± 85 at six-tenths of the ultimate concentration due to gravity alone. While this agreement leaves much to be desired, comparing the experimental results with results from the relationships of other investigators, equations 13 through 18, will show that none, with the possible exception of equation 15, can express all of the data so well. Equation 15, requiring three unknown constants, could not be checked.

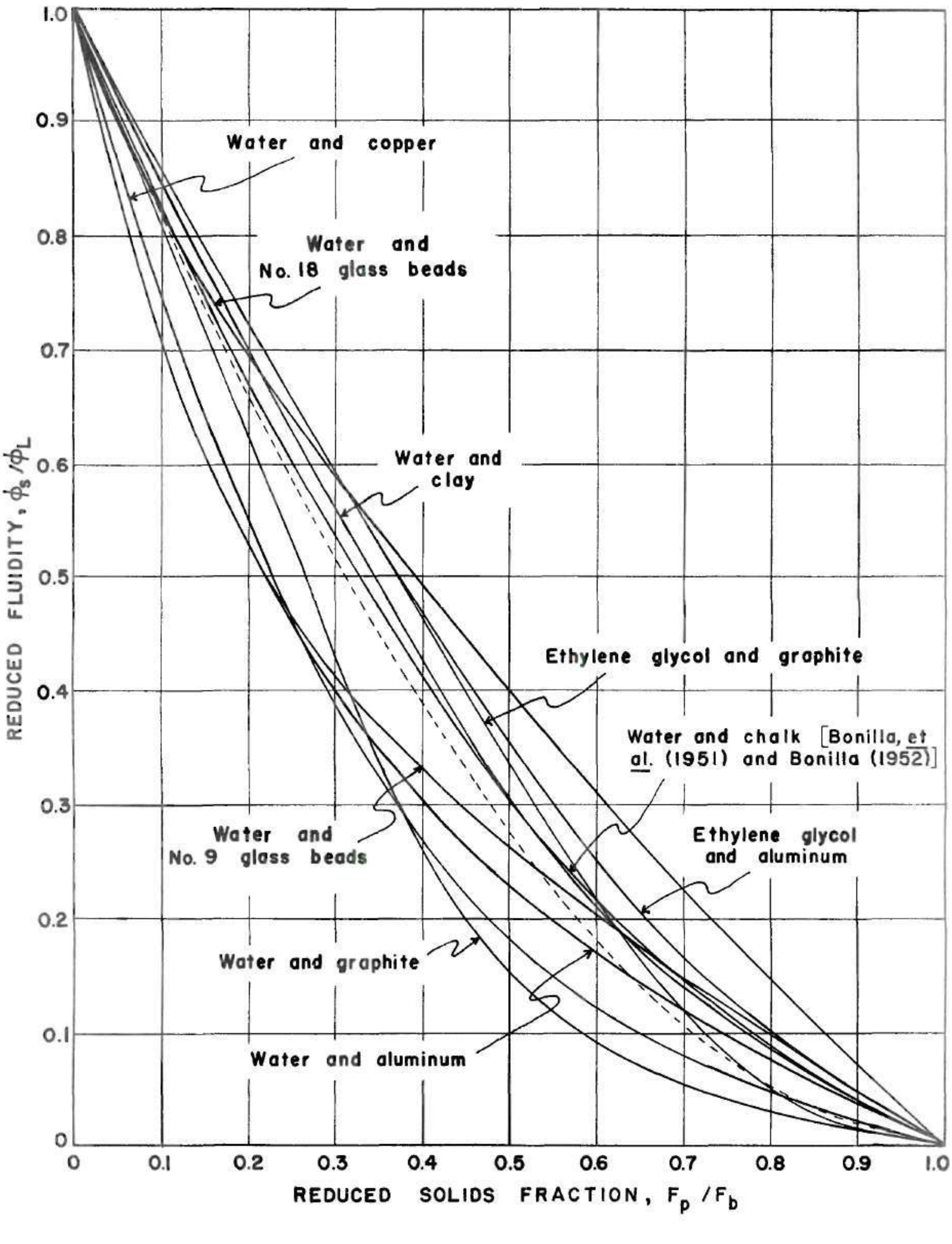


Figure 31. Reduced Plot of Fluidity Data. (The broken line corresponds to equation 35.)

The effect of dispersants may be considered in the light of equation 35, for possibly the greatest effect is exerted through viscosity. Dispersants, when effective, increase the volume concentration of the solid material in the bed that results from the gravity sedimentation of a suspension. To be sure, the rate of sedimentation under such conditions is decreased, but the compaction is ultimately increased. Since F_b is increased, the apparent viscosity of a suspension as indicated by equation 35 would be decreased. This result was observed in the viscosity investigation. That the addition of a fatty acid or an aluminum soap to dispersions of polar solids such as SiO_2, TiO_2 and inorganic pigments in oils increases the fluidity and decreases the volume occupied by the sediment has also been noted by Alexander (1950).

D. The Investigation of Bonilla, *et al.* (1951)

As mentioned in both the Preface and the Introduction, the only other investigation of the heat transfer properties of a suspension which sheds any light on the heat transfer mechanism is that of Bonilla, Cervi, Colven and Wang (1951). These investigators studied water-chalk suspensions containing up to 18 per cent chalk by weight flowing in a 1-1/2-inch horizontal pipe at rates of from 33,650 lb./hr. to 1330 lb./hr. The pipe was steam-jacketed, had thermocouples embedded in the wall, was preceded by a fluid-mixing chamber and was followed by another fluid-mixing chamber. The suspension's apparent viscosity was measured with a capillary flow meter quite similar to the one described in the preceding section on viscosity. In general, the investigation was quite comparable to the investigation described here.

Since a thorough description of the investigation and complete experimental data have been published, no tabulation of calculated values will be given here, but the results will be incorporated in their entirety in the correlations presented in the following section.

VI. CORRELATION OF RESULTS

A. The Correlation of Bonilla, et al. (1951)

A correlation between the average individual coefficient of heat transfer at a pipe wall with some of the properties of a water-solid suspension flowing inside the pipe was obtained by Bonilla, et al., as follows: Using the value for water alone for thermal conductivity, equation 14 to express viscosity and the weighted averages of the individual properties of the liquid and the solid for density and heat capacity, Reynolds, Prandtl and Nusselt numbers were evaluated. From a plot of $Nu/Pr^{1/3}$ versus weight per cent solid with Re as the parameter, it was found that the value of $Nu/Pr^{1/3}$ fell off roughly linearly with increasing concentration of the solid material. Accordingly, the relationship

$$\left(\frac{Nu}{Pr^{1/3}}\right)_S = \left(\frac{Nu}{Pr^{1/3}}\right)_L - 555W , \qquad (36)$$

where W is the fraction of solid material by weight composing the suspension, was found to express the result with moderate accuracy.

B. The Proposed Correlation

In all probability a correlation of this type, differing only in the value or values of the constant, could be obtained for any apparatus and any liquid-solid suspension. However, many investigations of single-phase systems point to the fact that a single correlation equally applicable to all suspensions should be possible. Such a correlation would surely be preferable. The information on the apparent properties of suspensions obtained in the auxiliary investigations reported here permits the exploration of this possibility.

Since all combinations of the many variables that enter into the heat transfer problem under discussion would lead to a multitude of relationships, it is only logical to make use of the arrangements of variables that have been found to apply to liquids. As discussed in the Introduction, a general equation applying to either heating or cooling which is relatively easy to evaluate has been developed for liquids. This equation, equation 2, rewritten here in terms of suspension properties, is

$$\frac{hD}{k_s} = 0.027 \left(\frac{Dv\rho_s}{\mu_s}\right)^{0.8} \left(\frac{C_{ps}\mu_s}{k_s}\right)^{1/3} \left(\frac{\mu}{\mu_w}\right)^{0.14}. \qquad (2)$$

Because some uncertainty in the apparent viscosity of a suspension is admitted and because the comparative unimportance of the solid's conductivity may thereby be shown, each of the main groups of equation 2 will be evaluated by several means; the group $(\mu/\mu_w)^{0.14}$ permits only one evaluation, and values of the ratio, without the exponent are, in fact, tabulated for each run in Table III. In the Nusselt number, hD/k, the heat transfer coefficient is an experimental quantity in this case and the pipe diameter is fixed, but the thermal conductivity of the suspension might be taken as that of the pure liquid, as was done by Bonilla, et al., or it might be evaluated by equation 9. The latter alternative, as shown by the thermal conductivity investigation, is correct. However, the group will be evaluated using the conductivity of water alone so that the difference of so doing may be shown. With the Reynolds number, $Dv\rho/\mu$, all the parameters may be regarded as established by the experimental conditions except viscosity for which many relationships have been offered; Reynolds numbers will therefore be evaluated using several of the expressions for viscosity given in the section devoted to the subject. The Prandtl number, $C_p\mu/k$, involving both conductivity and viscosity, will be evaluated using only the conductivity of a suspension as given by equation 9 and viscosity as given by the same equations used in evaluating the Reynolds number. The heat capacity in the Prandtl number, like the density of the Reynolds number, is an additive property and can only be evaluated as the weighted average of the properties of the components making up the suspension.

C. Sample Calculation and Results

Each of the groups is simply evaluated by proper substitution of values obtained directly from the tables, figures or previous calculations. However, to avoid any uncertainty and to complete the sample calculation for run No. 32 begun on page 34, a calculation of the individual groups comprising equation 2 will be included using several expressions for the various parameters as discussed above.

In Table III, h was given as 4940 Btu/hr.,ft^2.,^{o}F., the pipe diameter was given as 0.0411 ft. on page 39 and the conductivity of water at the bulk mean suspension temperature of 181.4^o F. may be found from a figure in the Appendix to be 0.397 Btu/hr.,ft^2. (^{o}F. per ft.). Therefore, if a Prandtl number were evaluated using this data, a value of

$$\frac{hD}{k_L} = \frac{4940 \text{ x } 0.0411}{0.397} = 512$$

would be obtained. If the conductivity of the suspension instead of the liquid were to be used, the suspension's conductivity could be evaluated directly from equation 9 or from the conductivity ratio given in Table III, which amounts to using equation 9. The value of the Prandtl number would then be

$$\frac{hD}{k_s} = \frac{4940 \text{ x } 0.0411}{0.397 \text{ x } 1.035} = 494 \text{ .}$$

Reynolds numbers may be calculated using each of the viscosity values, the calculations for which were begun on page 106. The velocity of flow, calculated on page 45, is 14.6 ft./sec., while the pipe diameter, 0.0411 ft., is given above. The suspension density at the temperature of the run was 69.9 lb-mass/ft.^{3} x 0.970/0.997 or 68.0 lb-mass/ft.^{3}, as explained on page 44. Therefore, using subscripts to indicate the relation by which viscosity was evaluated, the following results are obtained:

$$\left(\frac{Dv\rho}{\mu}\right)_{\text{friction factor}} = \frac{0.0411 \text{ x } 14.6 \text{ x } 68.0 \text{ x } 3600}{1.08} = 136{,}000 \text{ ,}$$

$$\left(\frac{Dv\rho}{\mu}\right)_{\text{liquid}} = \frac{0.0411 \text{ x } 14.6 \text{ x } 68.0 \text{ x } 3600}{0.833} = 176{,}000 \text{ ,}$$

$$\left(\frac{Dv\rho}{\mu}\right)_{\text{equation 16}} = \frac{0.0411 \text{ x } 14.6 \text{ x } 68.0 \text{ x } 3600}{0.990} = 148{,}000 \text{ ,}$$

$$\left(\frac{Dv\rho}{\mu}\right)_{\text{equation 14}} = \frac{0.0411 \times 14.6 \times 68.0 \times 3600}{1.38} = 106{,}000 \ ,$$

$$\left(\frac{Dv\rho}{\mu}\right)_{\text{experimental}} = \frac{0.0411 \times 14.6 \times 68.0 \times 3600}{1.07} = 137{,}000 \ ,$$

$$\left(\frac{Dv\rho}{\mu}\right)_{\text{equation 35}} = \frac{0.0411 \times 14.6 \times 68.0 \times 3600}{1.05} = 140{,}000 \ .$$

In the calculation of Prandtl numbers the viscosities used in the Reynolds numbers calculations above and the conductivity of the suspension, 0.397 Btu/hr.,ft.^{2}(°F.per ft.) x 1.035 or 0.411 Btu/hr.,ft.^{2}(°F. per ft.) will be used. The heat capacity of the suspension is given as 0.868 Btu/lb-mass,°F. on page 40. Therefore, again using subscripts to indicate the appropriate viscosity source,

$$\left(\frac{C_p\mu}{k_s}\right)_{\text{friction factor}} = \frac{0.868 \times 1.11}{0.411} = 2.34 \ ,$$

$$\left(\frac{C_p\mu}{k_s}\right)_{\text{liquid}} = \frac{0.868 \times 0.833}{0.411} = 1.76 \ ,$$

$$\left(\frac{C_p\mu}{k_s}\right)_{\text{equation 16}} = \frac{0.868 \times 0.990}{0.411} = 2.09 \ ,$$

$$\left(\frac{C_p\mu}{k_s}\right)_{\text{equation 14}} = \frac{0.868 \times 1.38}{0.411} = 2.91 \ ,$$

$$\left(\frac{C_p\mu}{k_s}\right)_{\text{experimental}} = \frac{0.868 \times 1.07}{0.411} = 2.26 \ ,$$

$$\left(\frac{C_p\mu}{k_s}\right)_{\text{equation 35}} = \frac{0.868 \times 1.05}{0.411} = 2.22 \ .$$

Each of these values is recorded in Table VII; plots of corresponding values, related and raised to the power indicated by equation 2, are shown in Figures 32 through 37. In addition, the data of Bonilla, et al. calculated identically, are plotted on the same figures. The solid lines on the figures represent equation 2.

TABLE VII

CALCULATED DIMENSIONLESS GROUPS

	Nusselt Number Using		Reynolds Number x 10^{-4} Using for Viscosity						Prandtl Number Using Equation 9 For Conductivity and Using for Viscosity					
Run No.	Conductivity of Liquid	Equation 9	Fanning Friction Factor Result	Viscosity of Liquid	Equation 16	Equation 14	Smoothed Experimental Data	Equation 35	Fanning Friction Factor Result	Viscosity of Liquid	Equation 16	Equation 14	Smoothed Experimental Data	Equation 35
1	393	393	14.0	11.6	11.6	11.6	11.6	11.6	1.96	2.35	2.35	2.35	2.35	2.35
2	341	337	13.4	9.04	8.97	7.69	8.85	8.76	1.53	2.27	2.28	2.67	2.36	2.34
3	362	349	7.79	8.73	8.43	6.66	7.79	7.61	2.33	2.19	2.49	2.87	2.46	2.51
4	375	355	7.60	9.20	8.71	6.60	7.60	7.33	1.60	2.06	2.49	2.85	2.48	2.57
5	330	306	8.40	9.20	8.53	6.40	7.07	6.77	2.16	1.97	2.35	2.83	2.56	2.67
6	458	458	25.3	14.3	14.3	14.3	14.3	14.3	1.30	2.31	2.31	2.31	2.31	2.31
7	295	295	9.40	7.68	7.68	7.68	7.68	7.68	2.16	2.64	2.64	2.64	2.64	2.64
8	451	451	17.0	12.6	12.6	12.6	12.6	12.6	3.36	2.67	2.67	2.67	2.67	2.67
9	487	465	17.4	13.2	13.1	11.9	13.1	13.2	4.05	2.57	2.60	2.86	2.60	2.58
10	255	254	6.90	5.81	5.75	5.23	5.77	5.77	2.41	2.85	2.88	3.16	2.87	2.87
11	479	477	18.0	13.7	13.5	12.4	13.7	13.7	1.78	2.33	2.36	2.59	2.34	2.34
12	281	280	11.4	5.47	5.42	4.89	5.46	5.46	1.20	2.50	2.53	2.80	2.51	2.51
13	362	361	16.2	9.45	9.36	8.44	9.43	9.43	1.45	2.44	2.47	2.74	2.45	2.45
14	509	508	20.3	17.5	17.3	15.6	17.4	17.4	1.83	2.11	2.14	2.37	2.22	2.13
15	548	545	20.4	18.2	18.0	16.1	18.0	18.1	1.84	2.06	2.08	2.33	2.07	2.07
16	325	323	9.10	8.91	8.80	7.88	8.82	8.86	2.22	2.49	2.51	2.81	2.51	2.50
17	532	502	25.5	18.6	17.6	13.5	16.8	17.3	1.29	1.78	1.87	2.44	1.96	1.90
18	413	390	13.0	12.1	11.5	8.85	10.9	11.3	1.59	1.70	1.79	2.33	1.88	1.82
19	523	494	23.0	19.9	19.0	14.6	18.0	18.7	1.53	1.78	1.87	2.42	1.96	1.89
20	710	672	40.0	21.4	20.4	15.7	19.4	20.1	0.872	1.63	1.71	2.22	1.80	1.74
21	581	550	25.1	17.9	17.1	13.2	16.3	16.9	1.18	1.65	1.74	2.25	1.81	1.75
22	722	644	34.0	19.5	17.6	12.9	15.9	17.0	0.781	1.36	1.51	2.06	1.67	1.56
23	474	422	23.0	16.5	15.2	11.2	13.7	14.8	1.18	1.61	1.79	2.44	1.98	1.84
24	347	328	15.5	12.5	11.9	9.08	11.3	11.6	1.18	1.47	1.54	2.01	1.61	1.57
25	634	536	53.0	25.6	21.8	15.7	18.6	20.8	0.563	1.17	1.37	1.91	1.61	1.43
26	308	256	15.8	12.9	10.8	7.74	9.19	10.4	1.12	1.38	1.64	2.29	1.93	1.71
27	542	441	43.0	25.8	21.1	15.1	17.5	20.0	0.664	1.11	1.35	1.89	1.63	1.43
28	231	189	3.73	7.00	5.76	4.11	4.80	5.46	2.36	1.26	1.53	2.14	1.83	1.61
29	268	268	8.20	7.57	7.57	7.57	7.57	7.57	2.35	2.54	2.54	2.54	2.54	2.54
30	317	311	5.55	7.65	7.05	5.25	6.61	6.85	3.22	2.35	2.54	3.41	2.71	2.62
31	507	503	14.7	16.6	15.7	12.0	15.1	15.3	3.33	2.01	2.13	2.78	2.21	2.17
32	512	494	13.6	17.7	14.8	10.7	13.7	14.0	2.34	1.76	2.09	2.91	2.24	2.22
33	334	321	7.60	9.80	8.15	5.83	7.58	7.62	2.51	1.95	2.34	3.27	2.52	2.50
34	204	197	4.50	5.50	4.73	3.41	4.46	4.51	2.46	2.02	2.35	3.25	2.48	2.46
35	376	356	10.0	17.8	13.2	9.50	12.3	11.9	2.97	1.67	2.26	3.14	2.43	2.50
36	308	292	7.60	10.4	7.90	5.67	8.50	7.20	2.71	1.97	2.60	3.63	2.42	2.85
37	425	404	29.0	18.5	13.9	10.0	10.8	13.5	1.00	1.53	2.04	2.84	2.62	2.11
38	288	275	19.8	11.1	8.67	6.20	6.85	8.45	0.889	1.59	2.03	2.84	2.57	2.08
39	375	334	20.0	18.1	9.08	7.34	6.74	8.36	1.13	1.24	2.49	3.08	3.35	2.70
40	195	179	4.20	8.84	5.52	4.12	4.08	5.38	3.02	1.40	2.24	3.00	3.11	2.36

(Continued)

TABLE VII (Concluded)

CALCULATED DIMENSIONLESS GROUPS

Run No.	Nusselt Number Using: Conductivity of Liquid	Nusselt Number Using: Equation 9	Reynolds Number x 10^{-4} Using for Viscosity: Fanning Friction Factor Result	Reynolds: Viscosity of Liquid	Reynolds: Equation 16	Reynolds: Equation 14	Reynolds: Smoothed Experimental Data	Reynolds: Equation 35	Prandtl Number Using Equation 9 For Conductivity and Using for Viscosity: Fanning Friction Factor Result	Prandtl: Viscosity of Liquid	Prandtl: Equation 16	Prandtl: Equation 14	Prandtl: Smoothed Experimental Data	Prandtl: Equation 35
41	352	313	26.0	20.5	9.84	8.08	7.16	8.97	0.832	1.06	2.20	2.67	3.02	2.41
42	291	291	8.20	8.66	8.66	8.66	8.66	8.66	2.57	2.43	2.43	2.43	2.43	2.43
43	302	290	7.60	9.17	8.73	6.73	7.97	8.04	2.72	2.26	2.37	3.08	2.59	2.57
44	291	255	8.10	9.40	8.29	6.01	6.24	6.66	1.63	1.74	1.97	2.72	2.62	2.46
45	416	364	19.5	16.4	14.5	10.5	11.9	11.6	1.37	1.63	1.84	2.54	2.45	2.30
46	412	361	20.2	14.6	12.9	9.34	9.68	10.4	1.37	1.89	2.15	2.97	2.86	2.67
47	468	383	22.6	15.3	12.6	9.02	7.55	8.82	1.13	1.67	2.03	2.84	3.40	2.91
48	287	234	7.70	9.65	7.90	5.65	4.66	5.50	2.05	1.64	2.00	2.79	3.38	2.87
49	197	161	2.15	5.03	4.12	2.95	2.41	2.87	4.58	1.96	2.39	3.33	4.09	3.44
50	667	520	32.0	18.4	14.3	10.2	6.91	8.93	0.815	1.41	1.82	2.51	3.77	2.92
51	437	346	22.6	19.7	15.3	10.9	7.44	9.52	1.20	1.49	1.78	2.49	3.66	2.86
52	345	256	19.2	13.0	9.54	6.87	3.59	5.23	0.897	1.32	1.81	2.51	4.80	3.29
53	210	155	3.55	6.41	4.70	3.38	1.77	2.57	2.62	1.45	1.97	2.74	5.25	3.60
54	352	238	11.8	13.9	9.12	5.13	1.82	3.65	1.38	1.18	1.79	3.19	8.97	4.48
55	598	406	--	18.5	12.2	6.90	2.36	4.92	--	1.10	1.67	2.95	8.64	4.14
56	385	242	28.2	17.3	10.3	7.78	1.06	2.88	0.629	1.02	1.73	2.28	16.7	6.17
57	150	93.7	10.0	7.11	4.16	3.18	0.414	1.14	0.950	1.34	1.71	2.99	23.0	8.35
58	329	192	9.60	16.5	9.10	7.08	0.633	1.85	1.66	0.98	1.75	2.25	25.2	8.63
59	207	123	7.20	7.92	4.34	3.39	0.263	0.862	0.978	1.24	2.26	2.89	37.2	10.3
60	296	296	4.30	5.99	5.99	5.99	5.99	5.99	3.20	2.29	2.29	2.29	2.29	2.29
61	316	303	16.2	12.2	11.8	9.23	8.25	9.70	1.52	2.01	2.10	2.67	2.98	2.54
62	419	400	11.6	15.4	14.8	11.6	13.2	12.2	2.66	2.01	2.09	2.67	2.98	2.54
63	261	245	9.80	5.03	4.76	3.64	2.85	3.56	1.16	2.26	2.39	3.12	3.98	3.19
64	455	428	--	15.8	15.0	11.5	8.93	11.3	--	2.09	2.21	2.89	3.68	2.94
65	389	348	--	15.6	14.2	10.4	6.11	7.92	--	1.89	2.10	2.85	4.84	3.74
66	256	228	--	9.15	8.29	6.08	3.58	4.62	--	2.08	2.30	3.13	5.32	4.11
67	354	321	--	14.7	13.4	9.90	6.18	8.02	--	1.93	2.12	2.87	4.60	3.55
68	198	179	--	7.02	6.39	4.72	2.95	3.83	--	2.08	2.29	3.09	4.94	3.82
69	308	264	--	14.0	12.1	8.74	3.88	4.97	--	1.80	2.09	2.90	6.53	5.09
70	156	134	--	6.60	5.71	4.11	1.83	2.34	--	1.91	2.21	3.06	6.90	5.39
71	363	363	--	11.8	11.8	11.8	11.8	11.8	--	2.29	2.29	2.29	2.29	2.29
72	311	311	--	8.53	8.53	8.53	8.53	8.53	--	2.47	2.47	2.47	2.47	2.47
73	126	126	--	2.80	2.80	2.80	2.80	2.80	--	2.67	2.67	2.67	2.67	2.67
74	151	151	0.890	0.940	0.940	0.940	0.940	0.940	49.6	47.0	47.0	47.0	47.0	47.0
75	181	152	0.750	1.08	0.906	0.649	0.742	0.593	46.6	32.3	38.5	53.8	47.1	58.9
76	178	133	1.07	1.18	0.882	0.634	0.603	0.405	27.9	25.3	34.0	47.4	49.8	73.8
77	83.9	63.0	1.12	0.566	0.496	0.356	0.337	0.293	21.3	31.9	42.9	59.9	63.0	72.6
78	108	96.5	1.12	0.832	0.751	0.551	0.382	0.301	26.0	35.0	38.8	52.9	76.2	96.9
79	94.0	77.8	0.790	0.957	0.798	0.571	0.155	0.088	34.5	27.5	33.1	46.1	170.0	299

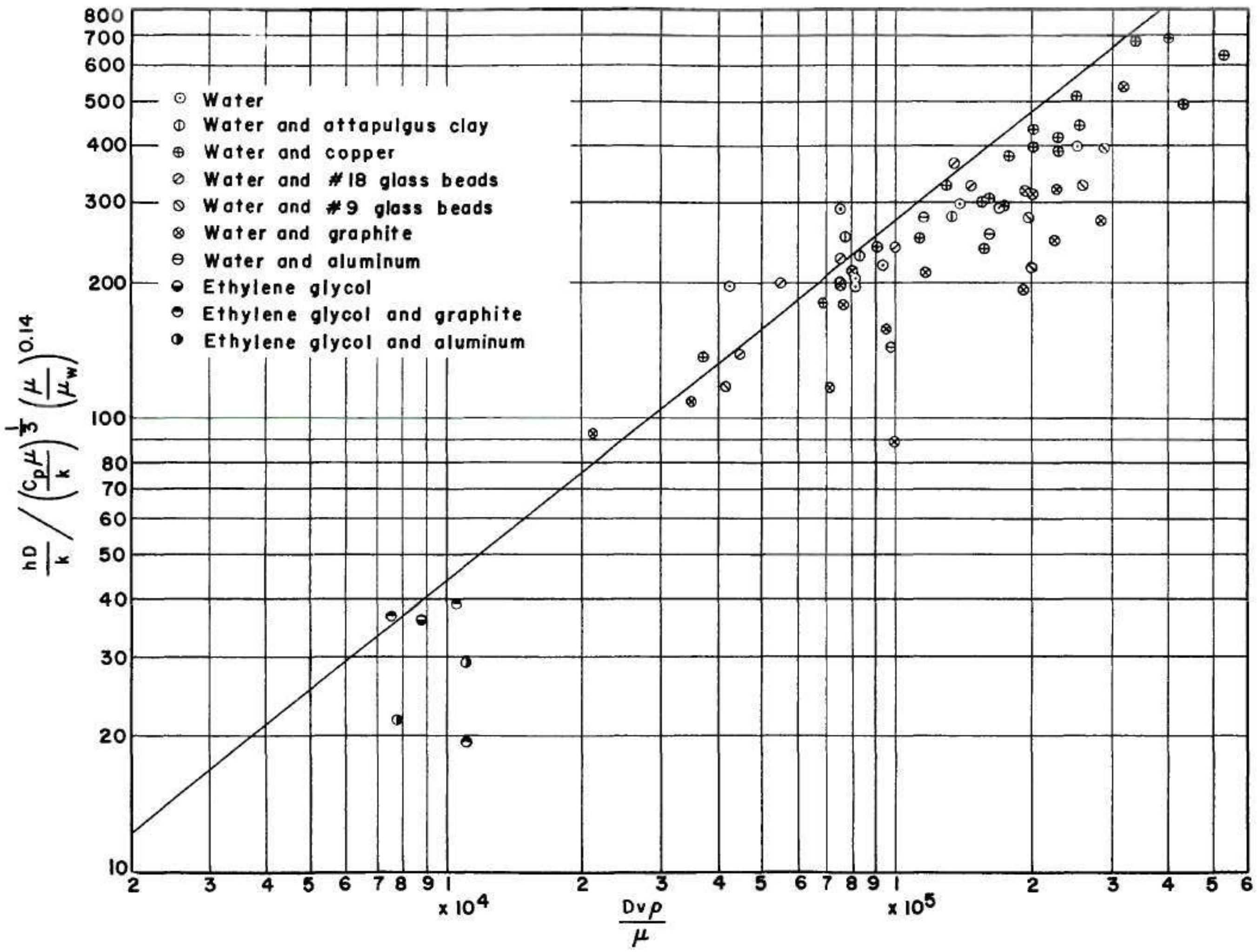


Figure 32. Calculated Heat Transfer Results Using Thermal Conductivity as Given by Equation 9 and Viscosity as Indicated by Fanning Friction Factor Results. (The solid line represents equation 2.)

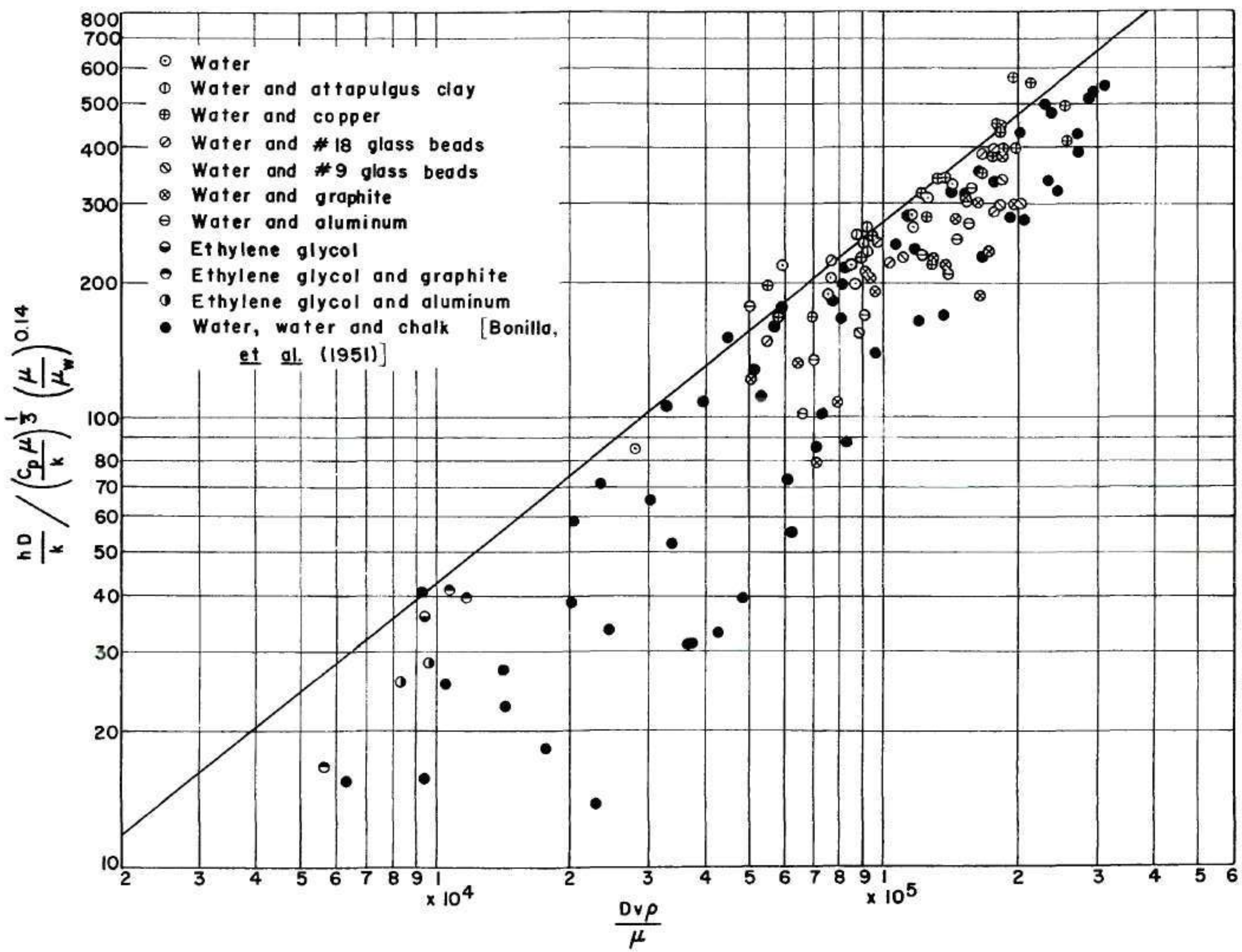


Figure 33. Calculated Heat Transfer Results Using Thermal Conductivity as Given by Equation 7 and the Viscosity of the Pure Liquid. (The solid line represents equation 2.)

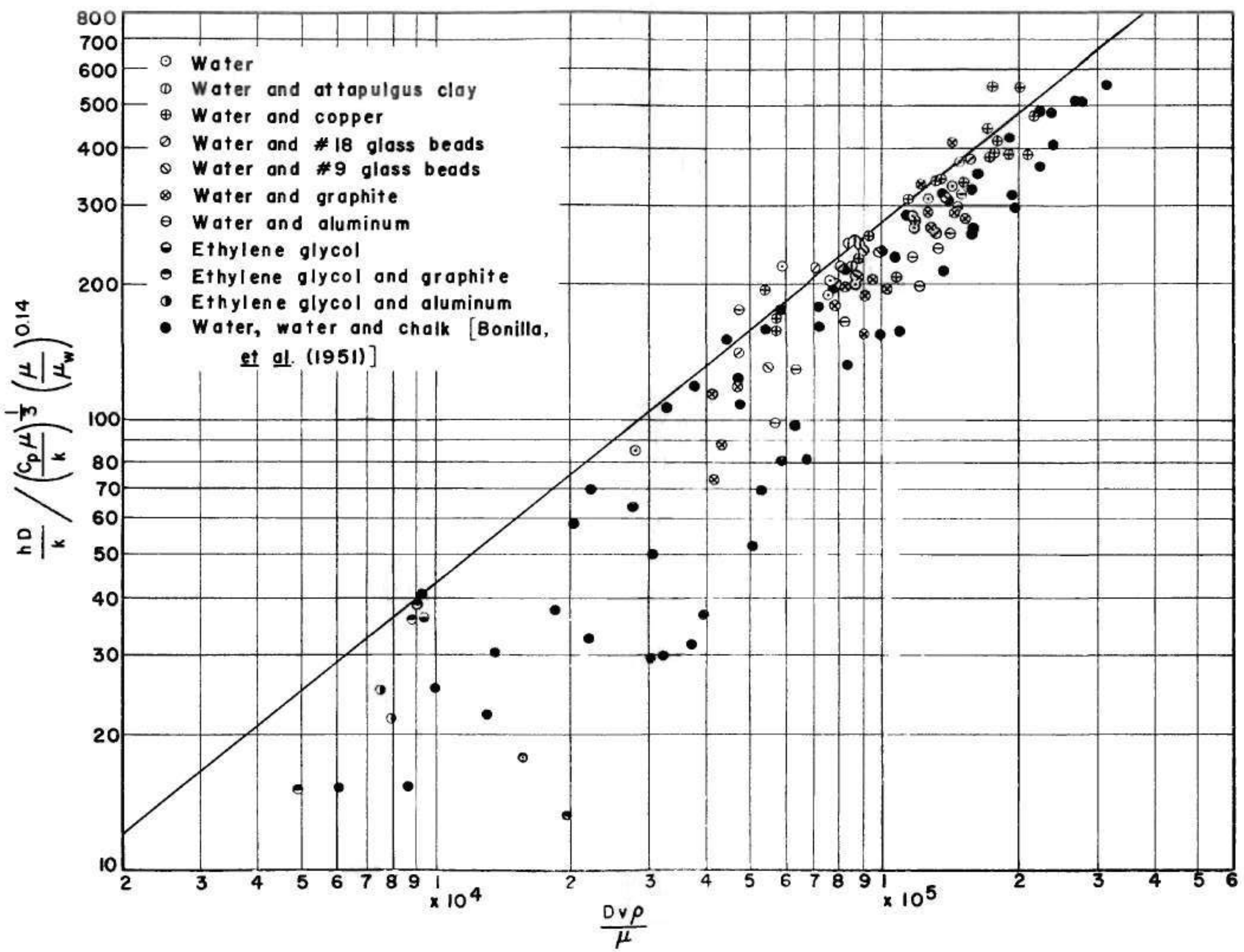


Figure 34. Calculated Heat Transfer Results Using Thermal Conductivity as Given by Equation 9 and Viscosity as Indicated by Equation 16. (The solid line represents equation 2.)

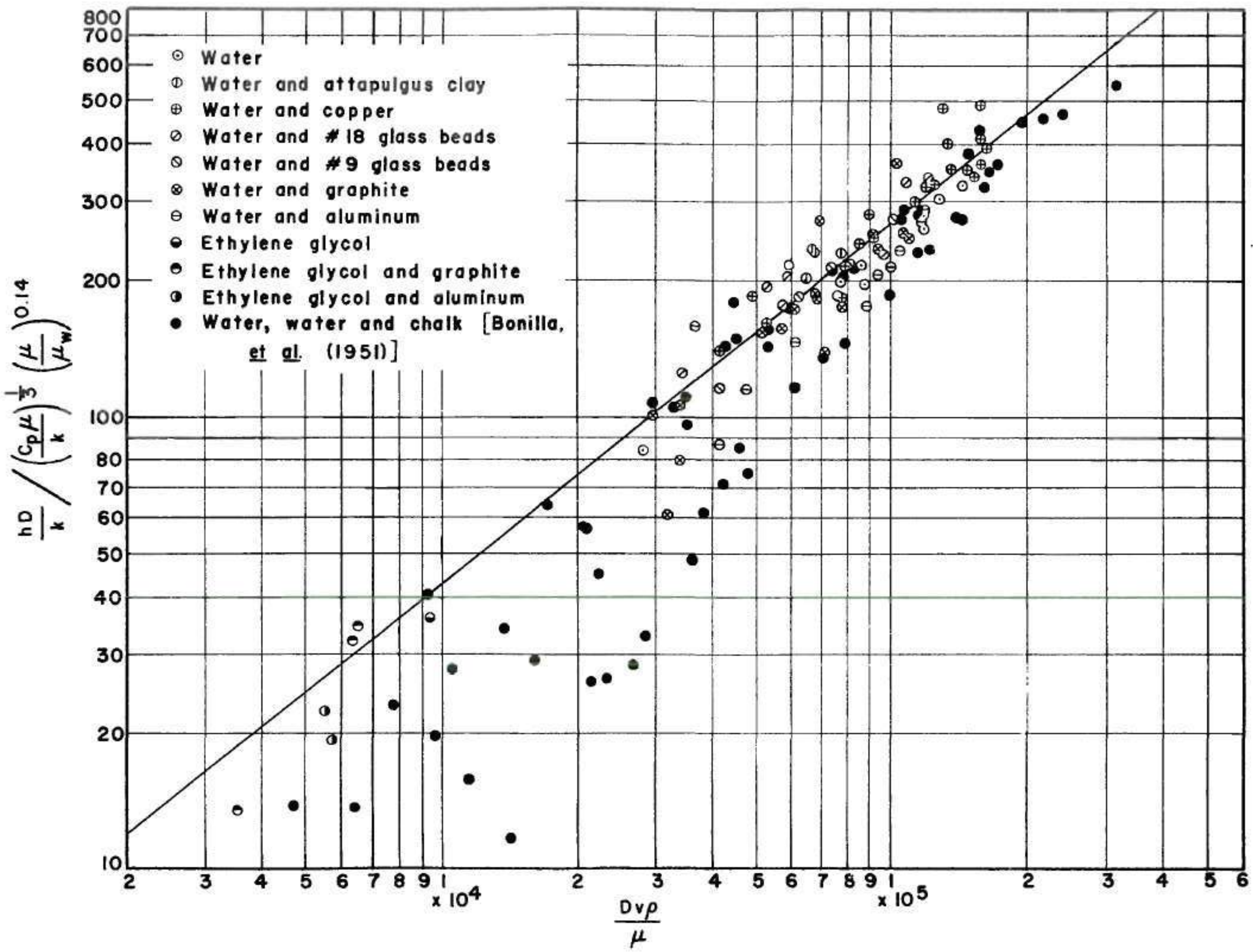


Figure 35. Calculated Heat Transfer Results Using Thermal Conductivity as Given by Equation 9 and Viscosity as Indicated by Equation 14. (The solid line represents equation 2.)

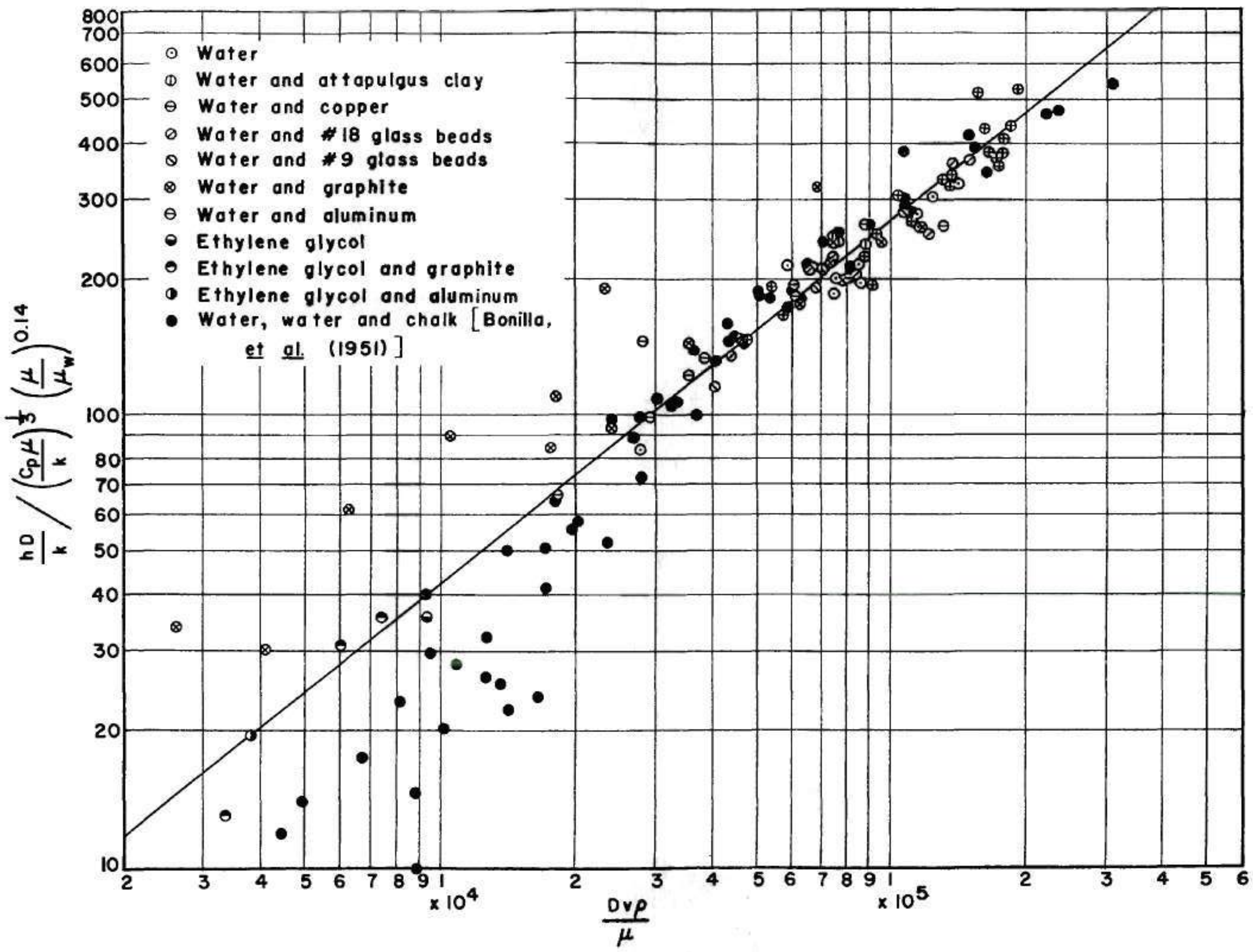


Figure 36. Calculated Heat Transfer Results Using Thermal Conductivity as Given by Equation 9 and Viscosity as Obtained from Experimental Measurements. (The solid line represents equation 2.)

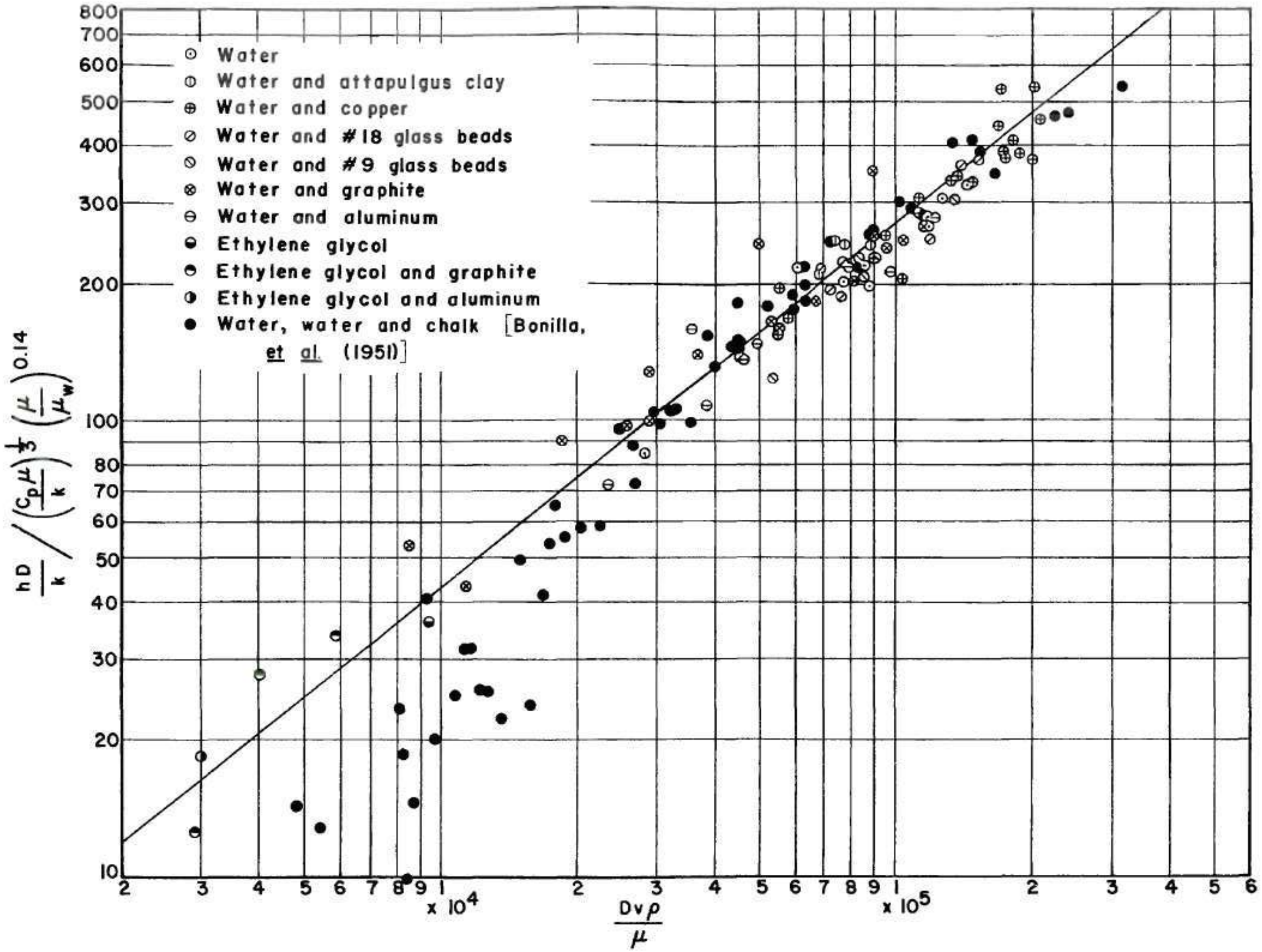


Figure 37. Calculated Heat Transfer Results Using Thermal Conductivity as Given by Equation 9 and Viscosity as Given by the Viscosity Correlation, Equation 35. (The solid line represents equation 2.)

V. DISCUSSION OF RESULTS

A. General Considerations

Since an analysis of results, limited to the data of that section, has been included in each of the experimental sections, only a cursory discussion of the experimental data is required here. The correlation requires more exhaustive treatment, however.

While other correlations were tested, e.g., correlations involving the Peclet number, to eliminate viscosity as a factor, or involving the Stanton number, to obviate conductivity, none of this type was found satisfactory, as was expected, since viscosity and conductivity are in reality factors requiring consideration. The so-called Dittus-Boelter equation, equation 1, was also completely evaluated and the results were examined. As discussed in the Introduction, this equation has been found suitable for liquids with viscosities not exceeding twice that of water and, hence, might be expected to include dilute suspensions. Because none of the deviations from any of the viscosity relationships are serious at low concentrations, equation 1 might be further expected to correlate low-concentration data using any of the viscosity relationships. This was found to be the case, but at higher concentrations deviations using all viscosity relationships were found to be serious. These latter results have not been included because a more satisfactory correlation was obtained using a similar expression, equation 2.

Examination of Figures 32 through 37 shows that all relationships are not equally good. In Figure 32 the viscosity used was that indicated by the pressure drop measurements made at the same time the heat transfer measurements were made; similar measurements were not reported by Bonilla, et al., nor were they made for every run of this investigation. It is obvious from the data available, however, that the agreement with equation 2 is not good. As explained on page 46, others have used pressure drop measurements in the exact manner of use here to correlate the flow properties of suspensions at rather high rates of flow. The viscosity obtained from this type of measurement is an apparent viscosity for turbulent flow and is quite different from the viscosity of laminar flow in the case of a suspension. The apparent viscosity for the turbulent flow of a suspension is less--sometimes much less--than the viscosity indicated in laminar flow; the data of Figure 32 generally indicate that a higher viscosity is required. Agreement with equation 2 should be expected in the case of pure liquids and dilute suspensions, since the viscosity in these cases is essentially independent of flow rate, i.e., they are Newtonian. As may be seen, the points for water, ethylene glycol and some suspensions fall near the solid line.

In Figure 33 through 37 all the data of this investigation, as well as the data of Bonilla, et al., are presented. It may readily be seen that the data fall considerably below the line representing equation 2 when the suspension's viscosity is used as that of the liquid or as that given by equations 14 or 16. When the viscosity as indicated by experimental measurements is used, as in Figure 36, the resulting points scatter about the expected line. But when the viscosity correlation is employed, as in Figure 37, all points, with the exception of a few water-chalk points of low flow rates, fall about the line of equation 2.

Equation 2, as pointed out above, is not recommended for use at a Reynolds number below 10,000. Most of the points which do not correlate are below this value, and hence their not correlating may be attributed in part at least to this fact. However, another factor may be involved. These data which do not correlate were obtained at low flow rates in a horizontal pipe. McAdams (1942) cites an investigation in which temperature explorations were made across a vertical diameter of a pipe inside which water flowed and which was steam-heated on the outside. At a Reynolds number of 11,200 the temperature distribution in the water was found to be far from symmetrical and was still somewhat unsymmetrical at a Reynolds number of 77,300. This result was apparently due to the rising of the heated water to the upper part of the pipe because of its reduced density. The situation undoubtedly existed in the water-chalk investigation and may have contributed to the discrepancies incurred at low Reynolds numbers.

It may be well to consider the range of variables covered by the correlation of Figure 37. The physical properties of the solid materials ranged from 513.8 lb-mass/ft.^{3} for the density of the copper powder to 124.2 lb-mass/ft.^{3} for the graphite's density, a conductivity from about 220 Btu/hr.,ft.^{2}(°F. per ft.) for the copper to 1.3 Btu/hr.,ft.^{2}(°F. per ft.) for the chalk, a heat capacity from about 0.22 Btu/lb-mass,°F. for graphite to about 0.093 Btu/lb-mass,°F. for copper, and a particle size from a median diameter of 260 microns for the largest glass beads to a maximum size of about 43 microns, indicative of a median size of the order of 2 to 5 microns, for the chalk. The liquids water and ethylene glycol differed in viscosity by a factor of about 10, in conductivity by a factor of about 3 and in heat capacity by a factor of almost 2. The pipes were nominal 3/8-inch and 1-1/2-inch I.P.S. Reynolds numbers from the lower limit of turbulent flow to 300,000 were employed. Both vertical and horizontal flow were investigated. Concentrations up to 45.7 per cent solid material by weight were used. In general, considerable coverage has been accomplished.

B. The Effect of Gravity Settling

The effect of settling, probably because a horizontal heat transfer section was used, was not considered in the investigation of the water-chalk suspensions. However, the vertical settling of the particles is worth considering here.

The case in which settling caused by gravity was greatest, a condition fulfilled by copper powder in water, will be taken for an example. The settling velocity will be calculated from the Stokes equation [for a discussion see Glasstone (1946)], which applies rigorously only to a spherical particle settling under conditions such that other particles in no way hinder the settling. Therefore, since the particles were neither spherical nor underwent unhindered settling, the actual settling rate will be somewhat less than calculated.

The Stokes equation may be written

$$v = 2.154 \times 10^{-9} \frac{gd^2 \,(\rho_p - \rho_L)}{\mu_L}\,, \tag{37}$$

when the settling velocity, v, is in ft./sec.; the acceleration of gravity, g, is in ft./sec.^{2}; the densities of the solid and the liquid, ρ_p and ρ_L, respectively, are in lb-mass/ft.^{3}; the viscosity of the liquid is in lb-mass/hr.,ft.; and the particle diameter, d, is in microns. The copper powder, as may be seen from the size distribution data of Figure 38 in the Appendix, had a median particle diameter, on the basis of number, of 3.3 microns. These data were obtained by the microscopic technique outlined by DallaValle (1948). The density of the copper powder, as used previously, was 513.8 lb-mass/ft.^{3} If a mean liquid temperature of 180° F. is taken, the density and viscosity of water will be found to be 60.5 lb-mass/ft.^{3} and 0.83 lb-mass/hr.,ft., respectively. Therefore,

$$v = \frac{2.154 \times 10^{-9} \times 32.14 \times (3.3)^2\ (513.8 - 60.5)}{0.83}$$

$$= 0.00041 \text{ ft./sec.}$$

As shown on Figure 38, the geometric standard deviation, σ_g, a measure of the particle size distribution, is equal to 2.27. From this value and from the median size on a number basis, DallaValle (1948) shows that the median particle diameter on the basis of weight may be obtained from the relationship

$$\log d_{weight} = \log d_{number} + 6.908 \log^2 \sigma_g \ . \qquad (38)$$

Therefore, the median particle diameter on the basis of weight is

$$\log d_{weight} = \log 3.3 + 6.908 \log^2 2.27$$

$$d_{weight} = 25 \text{ microns} \ .$$

The settling velocity of a copper particle of this size is only

$$v = \frac{2.15 \times 10^{-9} \times 32.14 \times (25)^2 \ (513.8 - 60.5)}{0.83}$$

$$= 0.024 \text{ ft./sec.}$$

Since the lowest bulk mean velocity for a copper suspension through the heat transfer section in any run was just less than five feet per second, it may be seen that the effect of settling was quite negligible in this investigation also.

C. Comparison of a Liquid with a Suspension

The question of how the heat transfer ability of a liquid compares with that of a suspension naturally arises. In the course of the experimental work a number of tests was made employing pure water. At the completion of three of these tests a quantity of powder was added to the water and, making no other changes, another test was made. Therefore, these results provide a direct comparison of the heat transfer merits of liquids and suspensions. These data are assembled in Table VIII.

Unfortunately, in all these cases the concentration of solid material was low. However, the data are consistent in showing an increase in the value of the heat transfer coefficient upon the addition.

Using the correlations for conductivity, viscosity and heat transfer, comparisons may be made for more extreme conditions. The question immediately arises as to what conditions may be made the bases for

TABLE VIII

COMPARISON OF HEAT-TRANSFERRING MERITS OF LIQUIDS AND SUSPENSIONS

Run Number			Measured Heat Transfer Coefficient (Btu/hr., ft.^{2}, °F.)	
Water	Suspension	Solid Material	Water	Suspension
8	9	Copper	4200	4560
29	30	Glass beads	2520	2970
42	43	Graphite	3170	3390

comparison--whether flow velocity, Reynolds number, pressure drop, pumping cost, etc. Since economic considerations would be beyond the scope of this discussion, comparisons will be limited to (1) identical average flow velocity, (2) identical Reynolds number and (3) identical pressure drop due to friction. Quite arbitrarily, water will be considered for the pure liquid and will be compared with a suspension of water and 10 volume per cent copper powder. It will be assumed that the same apparatus is used in the comparison and that the same bulk fluid temperature, 150° F., exists in each case. The properties of the liquid and solid materials as given in the figures and tables in the Appendix will be used.

For purposes of comparison, equation 2 may be employed, and, using it, the ratio of the average individual coefficients of heat transfer for the liquid and the suspension may be expressed as

$$\frac{h_s}{h_L} = \frac{\left(\dfrac{k^{0.67}\,\rho^{0.8}\,C_p^{\;0.33}}{\mu^{0.33}\,\mu_w^{\;0.33}}\right)_s}{\left(\dfrac{k^{0.67}\,\rho^{0.8}\,C_p^{\;0.33}}{\mu^{0.33}\,\mu_w^{\;0.14}}\right)_L} \tag{39}$$

if a pipe of the same diameter is employed and the same flow velocity is considered in both cases. If, for the suspension, density and heat capacity are evaluated as the weighted average of these properties for the individual components, if conductivity is evaluated by equation 9 and if viscosity is evaluated by equation 35, the ratio will be found to be

$$\frac{h_s}{h_L} = \frac{\left(\dfrac{0.450}{0.384}\right)^{0.67}\left(\dfrac{106}{61.2}\right)^{0.8}\left(\dfrac{0.562}{1.00}\right)^{0.33}}{\left(\dfrac{1.52}{1.05}\right)^{0.33}\left(\dfrac{\mu_{ws}}{\mu_{wL}}\right)^{0.14}} = \frac{1.27}{\left(\dfrac{\mu_{ws}}{\mu_{wL}}\right)^{0.14}} \; .$$

If the coefficients of heat transfer differ, the pipe wall temperatures will differ, and the viscosities at the wall will differ. The temperatures at the pipe wall will vary inversely as the heat transfer coefficients, and, while viscosity does not vary linearly with temperature, it may be assumed to do so for small temperature differences. Therefore, by a trial and error calculation

$$\frac{h_s}{h_L} = \frac{1.27}{\left(\dfrac{1.52}{1.05} \times \dfrac{1.00}{1.22}\right)^{0.14}} = 1.22 \; .$$

By similar calculations the liquid alone would be found to be about a 3 per cent better heat transfer agent when comparison is made at identical Reynolds numbers, while the suspension would be a slightly better agent on the basis of constant pressure drop.

VI. CONCLUSIONS

From this work three principal conclusions may be drawn:

1. The average individual coefficient of heat transfer between a pipe wall and a liquid-solid suspension flowing turbulently inside the pipe is given by the well-known Dittus-Boelter equation for dilute suspensions and by the relationship

$$\frac{hD}{k_s} = 0.027\left(\frac{Dv\rho_s}{\mu_s}\right)^{0.8} \quad \left(\frac{C_{ps}\mu_s}{k_s}\right)^{1/3} \quad \left(\frac{\mu}{\mu_w}\right)^{0.14}$$

for all concentrations studied if thermal conductivity and viscosity are evaluated by the equations below, if density and heat capacity are calculated as the weighted average of the values for the individual components and if other terms are evaluated in the usual fashion.

2. The thermal conductivity of a liquid-solid suspension is given by the relationship

$$k_s = k_L\left[\frac{2k_L + k_p - 2F(k_L - k_p)}{2k_L + k_p + F(k_L - k_p)}\right].$$

3. The viscosity of a liquid-solid suspension is described adequately for heat transfer purposes by the relationship

$$\mu_s = \frac{\mu_L}{\left(1 - \frac{F}{F_b}\right)^{1.8}}$$

4. Resistance to the turbulent flow of a suspension in a pipe may be predicted from a Fanning friction factor versus Reynolds number plot, provided that the viscosity of the pure liquid and the density of the suspension are used in evaluating the Reynolds number.

The results of the investigation emphasize the fact that the primary resistance to heat transfer between a pipe wall and a fluid flowing inside the pipe is the boundary layer and that viscosity, in particular, should be evaluated at the temperature and conditions of this layer.

Solely as a heat transfer medium, a suspension would seem to have little to recommend it, for the disadvantages arising from the difficulties of handling more than outweigh the advantages. The value of this investigation lies in the fact that relationships have been set forth by which a suspension may be treated when the use of a suspension is mandatory.

VII. BIBLIOGRAPHY

ALEXANDER, A. E.
1950. "Surface Chemistry and Colloids." Advances in Colloid Science, Vol. III. Interscience Publishers, Inc., New York.

ALVES, G. E.
1949. "Flow of Non-Newtonian Suspensions." Chemical Engineering 56, 107-109.

BINGHAM, E. C.
1922. Fluidity and Plasticity. McGraw-Hill Book Company, Inc., New York.

---- AND DURHAM, T. C.
1911. "The Viscosity and Fluidity of Suspensions of Finely-Divided Solids in Liquids." American Chemical Journal 46, 278-297.

BONILLA, C. F.
1952. Personal communication.

----, GERVI, A., JR., COLVEN, T. J., JR. AND WANG, S. J.
1951. "Heat Transfer to Slurries in Pipes. Chalk and Water in Turbulent Flow." Preprint, Heat Transfer Symposium, Forty-fourth Annual Meeting of the American Institute of Chemical Engineers, December.

CALDWELL, D. H. AND BABBITT, H. E.
1941. "The Flow of Muds, Sludges and Suspensions in Circular Pipe." Transactions of the American Institute of Chemical Engineers 37, 237-266; Industrial and Engineering Chemistry 33, 249-256.

CALDWELL. O. G. AND MARSHALL. C. E.
1942. A Study of Some Chemical and Physical Properties of the Clay Minerals Nontronite, Attapulgite and Saponite. University of Missouri, Agricultural Experiment Station Bulletin No. 354, Columbia.

CARBIDE AND CARBON CHEMICALS CORPORATION
1947. Glycols. New York.

COLBURN, A. P.
1933. "A Method of Correlating Forced Convertion Heat Transfer Data and a Comparison With Fluid Friction." Transactions of the American Institute of Chemical Engineers 29, 174-210.

DALLAVALLE, J. M.
1948. Micromeritics. Pitman Publishing Corporation, New York.

EINSTEIN, A.
1906. "Eine neue Bestimmung der Moleküldimensionen." Annalen der Physik 19, 289-306.
1911. "Berichtigung zu meiner Arbeit: Eine neue Bestimmung der Moleküldimensionen." Annalen der Physik 34, 591-592.

EIRICH, F., BUNZL, M. AND MARGARETHA, H.
1936. "Untersuchungen über die Viscosität von Suspensionen und Lösungen. 4. Uber die Viskosität von Kugelsuspensionen." Kolloid Zeitschrift 74, 276-285.

GLASSTONE, S.
1946. Textbook of Physical Chemistry. D. Van Nostrand Company, Inc., New York.

GUTH, E. AND SIMHA, R.
1936. "Untersuchungen über die Viskosität von Suspensionen und Lösungen. 3. Uber die Viskosität von Kugelsuspension." Kolloid Zeitschrift 74, 266-275.

HAGENBACH, E.
1860. "Ueber die Bestimmung der Zähigkeit einer Flüssigkeit durch den Ausflufs aus Röhren." Annalen Der Physik und Chemie 109, 385-426.

HALL, H. T. AND FUOSS, R. M.
1951. "Empirical Analysis of Viscosity Data." The Journal of the American Chemical Society 73, 265-269.

HATSCHEK, EMIL
1913. "The General Theory of Viscosity of Two-Phase Systems." Transactions of the Faraday Society 9, 80-92.
1920. "Die Viskosität von Blutkörperchen-Suspensionen." Kolloid Zeitschrift 27, 163-165.
1928. The Viscosity of Liquids. G. Bell and Sons, Ltd., London,

HEWITT, D. M.
1952. Personal communication.

HODGMAN, C. D. (Editor)
1946. Handbook of Chemistry and Physics. Chemical Rubber Publishing Co., Cleveland.

JACKSON, W.
1942. "The Thermal and Electrical Properties of Bitumastic Compounds Containing Quartz Sand." The Philosophical Magazine 33, 81-89.

KEENAN, J. H. AND KEYES, F. G.
1936. Thermodynamic Properties of Steam. John Wiley and Sons, Inc., New York.

LANGE, N. A. (Editor)
1949. Handbook of Chemistry. Handbook Publishers, Inc., Sandusky, Ohio.

McADAMS, W. H.
1942. Heat Transmission. McGraw-Hill Book Company, Inc., New York.

McBAIN, J. W.
1950. Colloid Science. D. C. Heath and Company, Boston.

MARTINELLI, R. C., SOUTHWELL, G. J., ALVES, G., CRAIG, H. L., WEINBERG, E. B., LANSING, N. F. AND BOELTER, L. M. K.
1942. "Heat Transfer and Pressure Drop for a Fluid Flowing in the Viscous Region Through a Vertical Pipe." Transactions of the American Institute of Chemical Engineers 38, 493-530.

MEISSNER, A.
1934. (a) "Isolierstoffe mit erhöhter Wärmeleitfähigkeit." Elektrotechnische Zeitschrift, 1193-1195, December 6.
(b) "Isolierstoffe mit erhöhter Wärmeleitfähigheit." Elektrotechnische Zeitschrift, 1218-1222, December 13.
1935. "Leistungssteigerung durch thermische Verbesserung der Isolierstoffe." Elektrotechnik und Maschinenbau, 289-293, June 23.

NORTON, F. H., JOHNSON, A. L. AND LAWRENCE, W. G.
1944. "Fundamental Study of Clay: VI. Flow Properties of Kaolinite-Water Suspensions." The Journal of the American Ceramic Society 27, 149-164.

PERRY, J. H. (Editor)
1950. Chemical Engineer's Handbook. McGraw-Hill Book Company, Inc., New York.

POWELL, R. W.
1937. "The Thermal and Electrical Conductivities of a Sample of Acheson Graphite from 0° C. to 800° C." The Proceedings of the Physical Society (London) 49, 419-426.

REINER, M.
1949. Deformation and Flow. H. K. Lewis and Co., Ltd., London.

ROBINSON, J. V.
1949. "The Viscosity of Suspensions of Spheres." The Journal of Physical and Colloid Chemistry 53, 1042-56.

ROESER, W. F. AND WENSEL, H. T.,
1941. "Methods of Testing Thermocouples and Thermocouple Materials." Temperature. Its Measurement and Control in Science and Industry. Reinhold Publishing Corporation, New York.

SIEDER, E. N. AND TATE, G. E.,
1936. "Heat Transfer and Pressure Drop of Liquids in Tubes." Industrial and Engineering Chemistry 28, 1429-1435.

TAREEF, B. M.
1940. "Thermal Conductivity of Colloidal Systems." Colloid Journal (U.S.S.R.) 6, 545-550. (Translated by E. L. Pollitzer.)

VAND, V.
1948. "Viscosity of Solutions and Suspensions. II. Experimental Determination of the Viscosity-Concentration Function of Spherical Suspensions." The Journal of Physical and Colloid Chemistry 52, 300-314.

WALKER, W. H., LEWIS, W. K., McADAMS, W. H. AND GILLILAND, E. R.
1937. Principles of Chemical Engineering. McGraw-Hill Book Company, Inc., New York.

WERT, R. W.
1952. Personal communications.

WILBERFORCE, L. R.
1891. "On the Calculation of the Coefficient of Viscosity of a Liquid from Its Rate of Flow Through a Capillary Tube." The Philosophical Magazine 31, 407-414.

VII. APPENDIX

TABLE IX

MISCELLANEOUS INFORMATION ON SUSPENSION MATERIALS

Material	Information
Attapulgus Clay	A mean particle diameter of between one and two microns was indicated by air permeametry according to the manufacturer, the Attapulgus Clay Company, Philadelphia 5, Pa. Its absolute thermal conductivity, estimated from its chemical composition as given by Caldwell and Marshall (1942) and its conductivity data as given by Lange (1949), was 7.5 Btu/hr., ft^2 (°F. per ft.). This latter value is probably good only as a first approximation. As far as this investigator is aware, only the thermal conductivity of bulk quantities has ever been measured. The absolute density of the material was found to be 158.3 lb./ft^3.
Copper Powder	The supplier was A. D. Mackay, 198 Broadway, New York, N. Y. The material had a measured absolute density of 513.8 lb./ft^3.
No. 18 Glass Beads	A median diameter of 35.0 microns, on a number basis, with a geometric standard deviation of 1.42 was indicated by optical measurements. The Minnesota Mining and Manufacturing Company, Saint Paul, Minnesota, was the manufacturer. Hewitt (1952) of the Minnesota Mining and Manufacturing Company reports that values for the heat capacity and thermal conductivity of 0.180 ± 0.01 Btu/lb., °F. and 0.67 Btu/hr., ft^2 (°F. per ft.), respectively, have been obtained. The material's absolute density was found to be 178.5 lb./ft^3.
No. 9 Glass Beads	A median diameter of 260 microns, on a number basis, and a geometric standard deviation of 1.27 were indicated. Otherwise, the No. 9 beads were identical to the No. 18 beads.
Acheson Graphite	The manufacturer, the National Carbon Company, Niagara Falls, N. Y., guaranteed that a minimum of 98.5 per cent by weight would pass a 200-mesh screen. A maximum ash content of 0.02 per cent by weight was specified for the material. The material was found to have an absolute density of 124.2 lb./ft^3.

(Continued)

TABLE IX (Concluded)

MISCELLANEOUS INFORMATION ON SUSPENSION MATERIALS

Material	Information
Aluminum Powder	A size distribution such that 75 per cent by weight would pass a 325-mesh screen with less than 5 per cent by weight being retained on a 100-mesh screen was specified by the manufacturer, the Aluminum Company of America, Pittsburgh, Pennsylvania. The absolute density of the powder was found to be 168.6 lb./ft.^{3}
Precipitated Chalk	According to the specifications given by Bonilla, et al. (1951), this material, a product of National Gypsum Company, Buffalo, N. Y., was 98.2 per cent $CaCO_3$, and 99.94 per cent (presumably by weight) of it would pass a 325-mesh screen. The heat capacity used by Bonilla, et al., 0.214 Btu/lb.,°F., is also that given by Hodgman (1946). A thermal conductivity for $CaCO_3$ of 1.3 Btu/hr.,ft.^{2}(°F. per ft.) at a temperature of 212° F. is given by Perry (1950). The material's density was 2.71 gm./cc. according to Bonilla, et al.

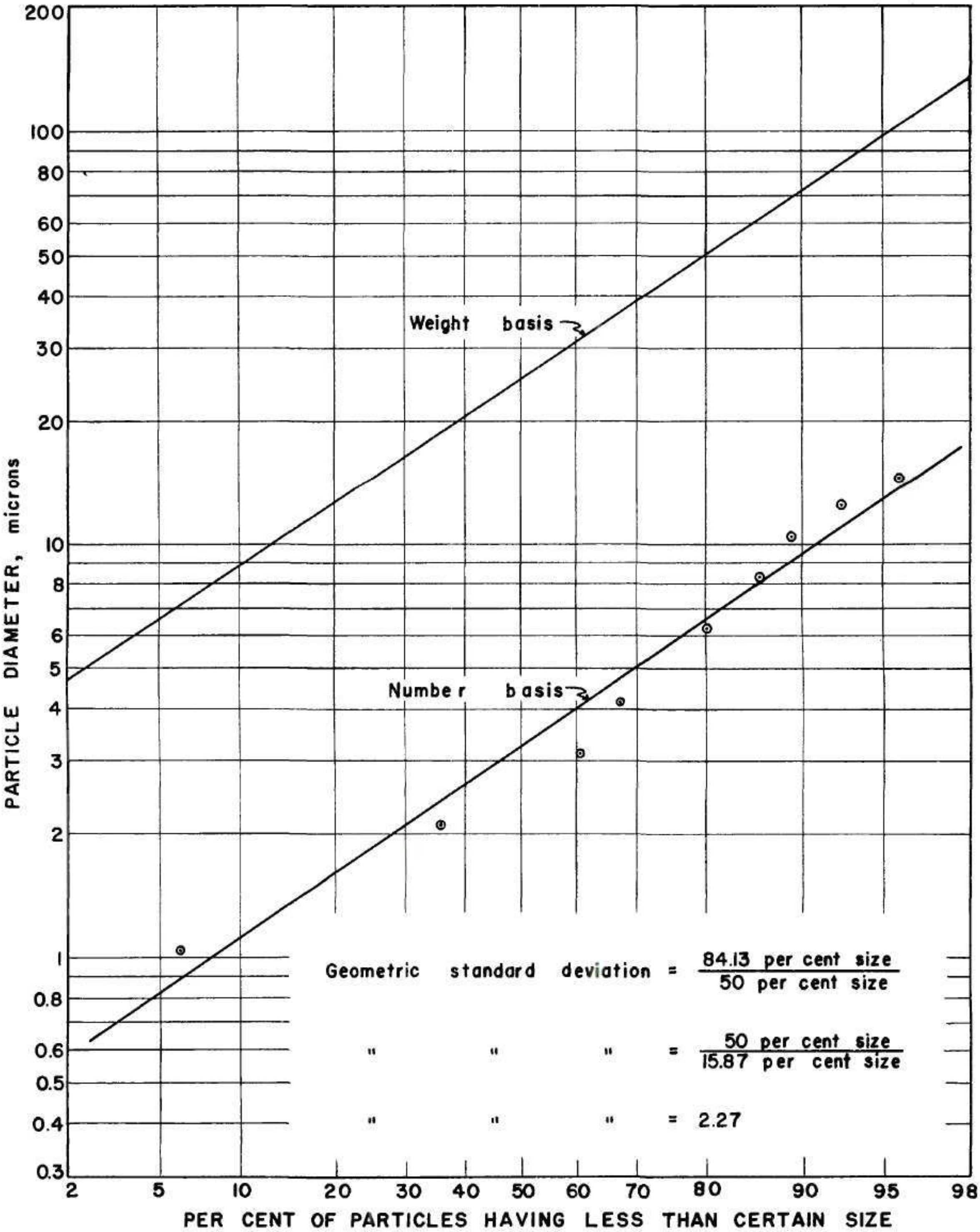


Figure 38. Size Distribution of Copper Powder.

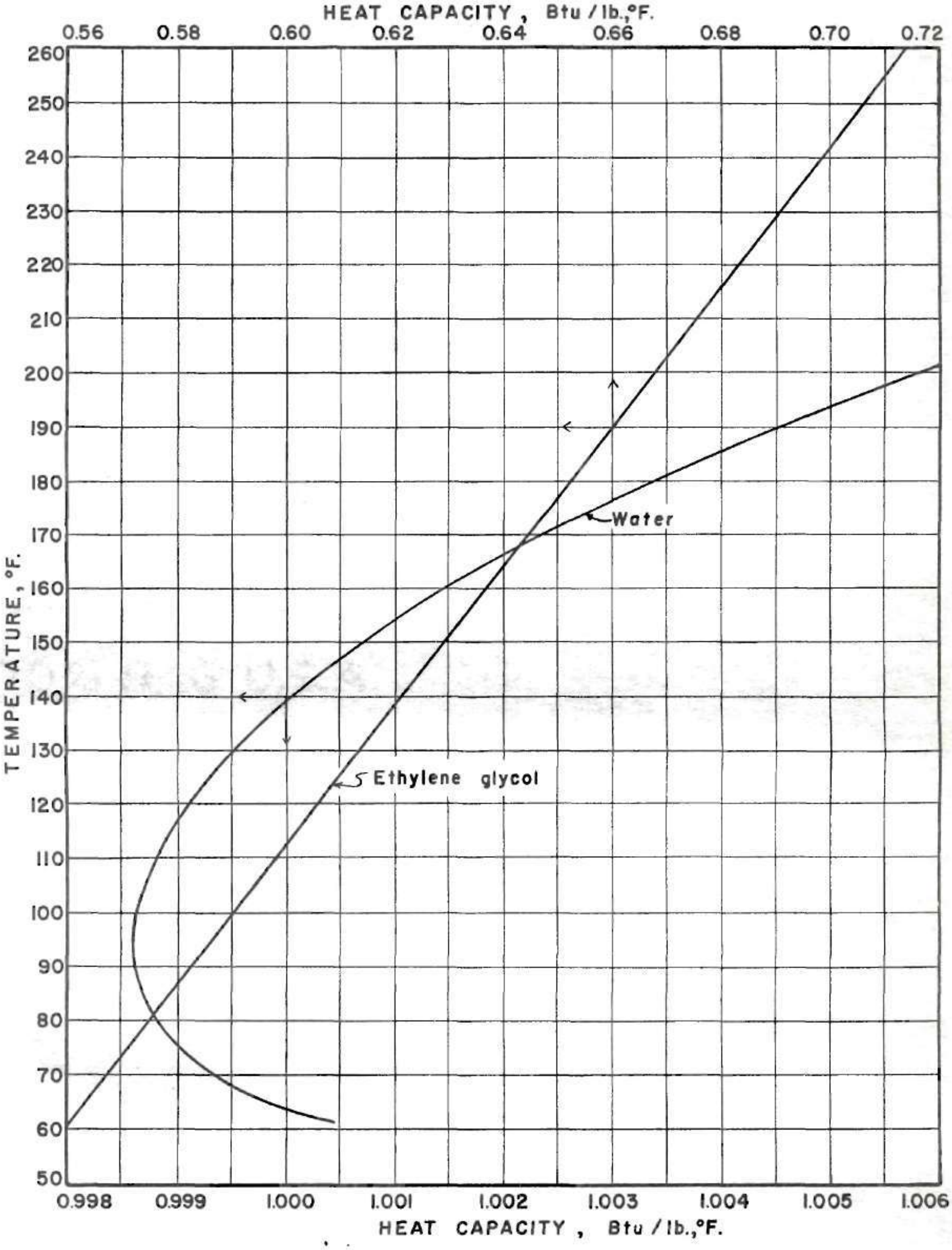


Figure 39. Heat Capacity of Water [Data from Perry (1950)] and of Ethylene Glycol [Data of Carbide and Carbon Chemicals Corporation (1947)] at Atmospheric Pressure.

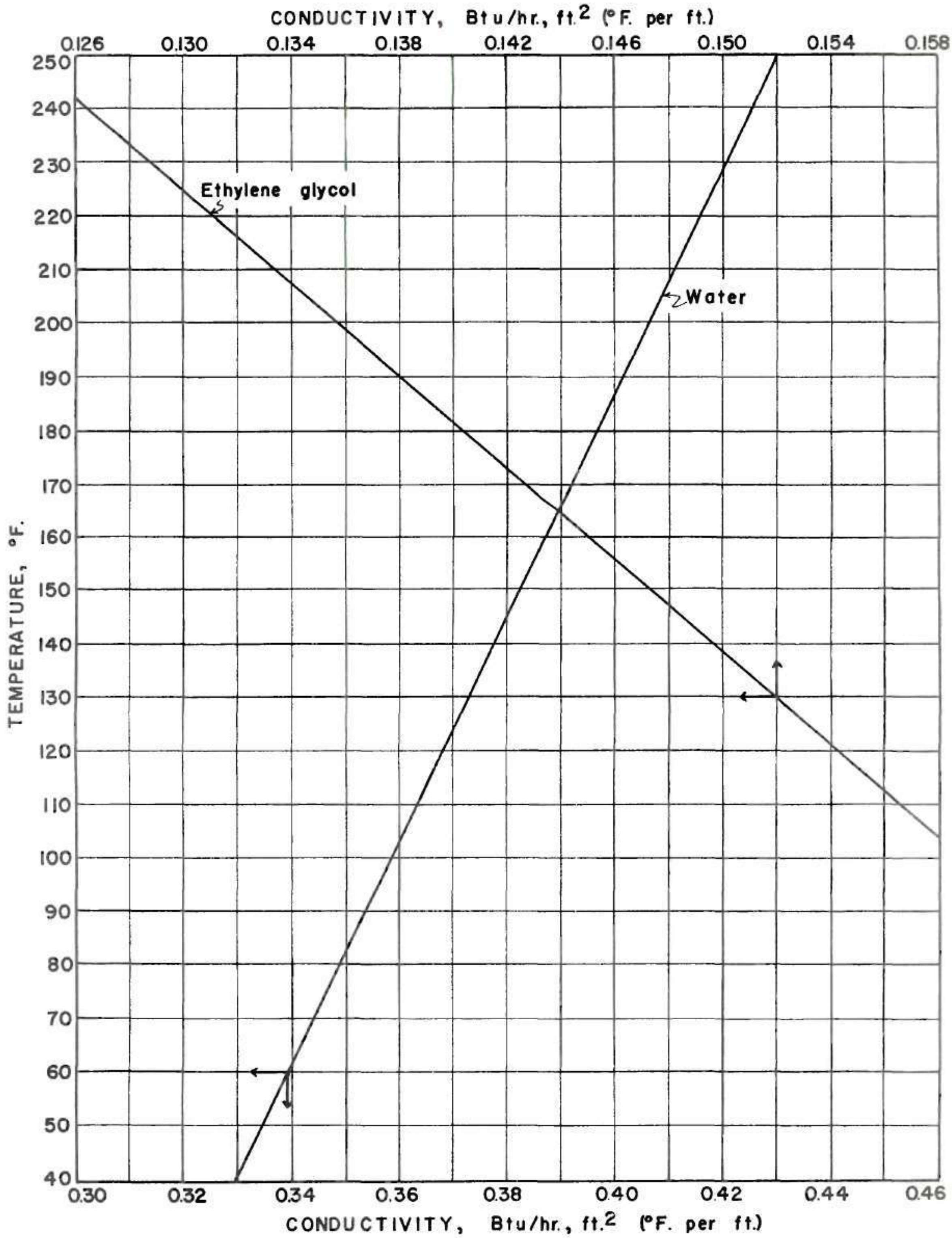


Figure 40. Thermal Conductivity of Water [Data from McAdams (1942)] and of Ethylene Glycol [Data of Carbide and Carbon Chemicals Corporation (1947)]. .

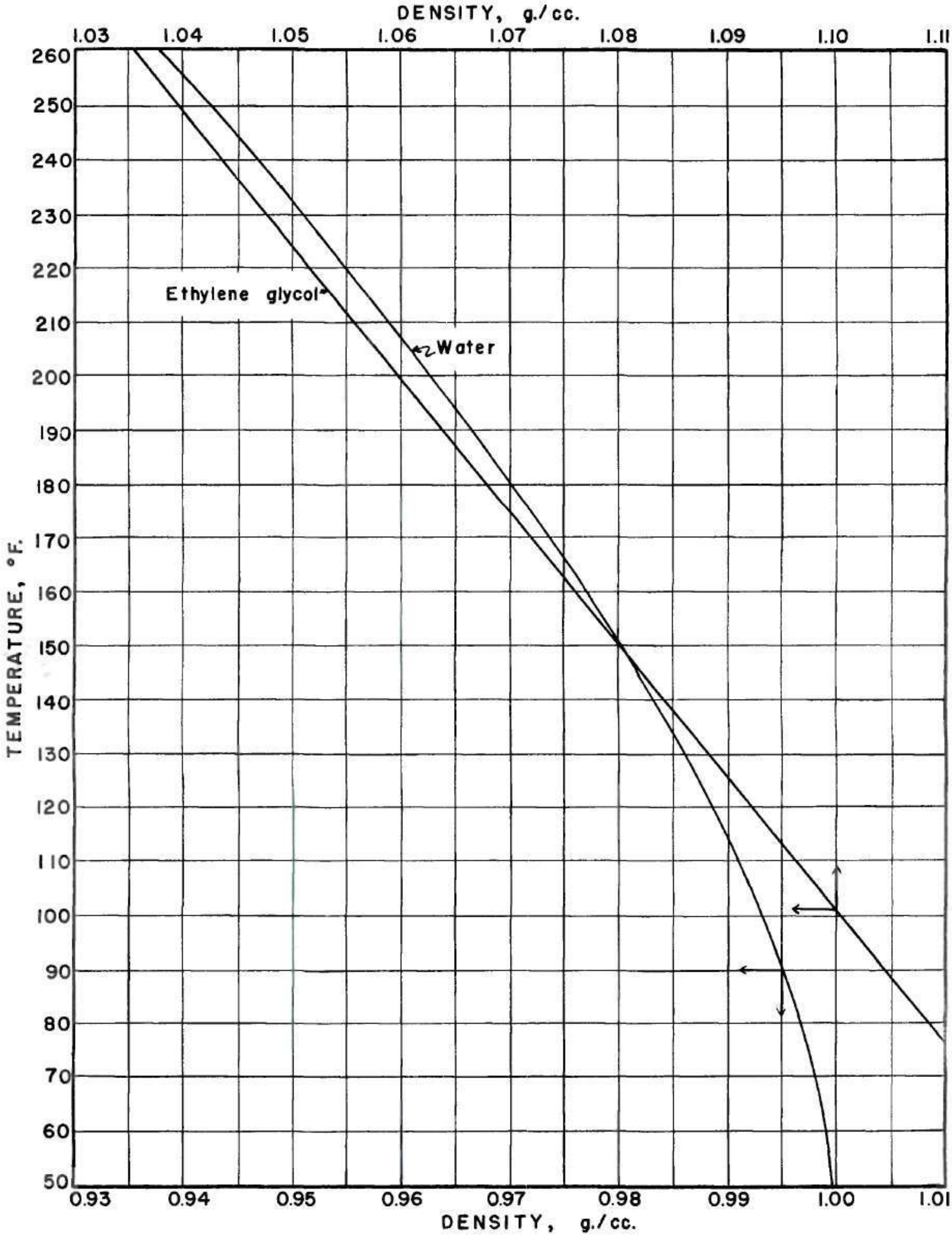


Figure 41. Density of Water (Based on 1.000 g./cc. = 62.43 lb./ft.^{3} at 4° C.) [Data from Hodgman (1946)] and of Ethylene Glycol [Data of Carbide and Carbon Chemicals Corporation (1947)].

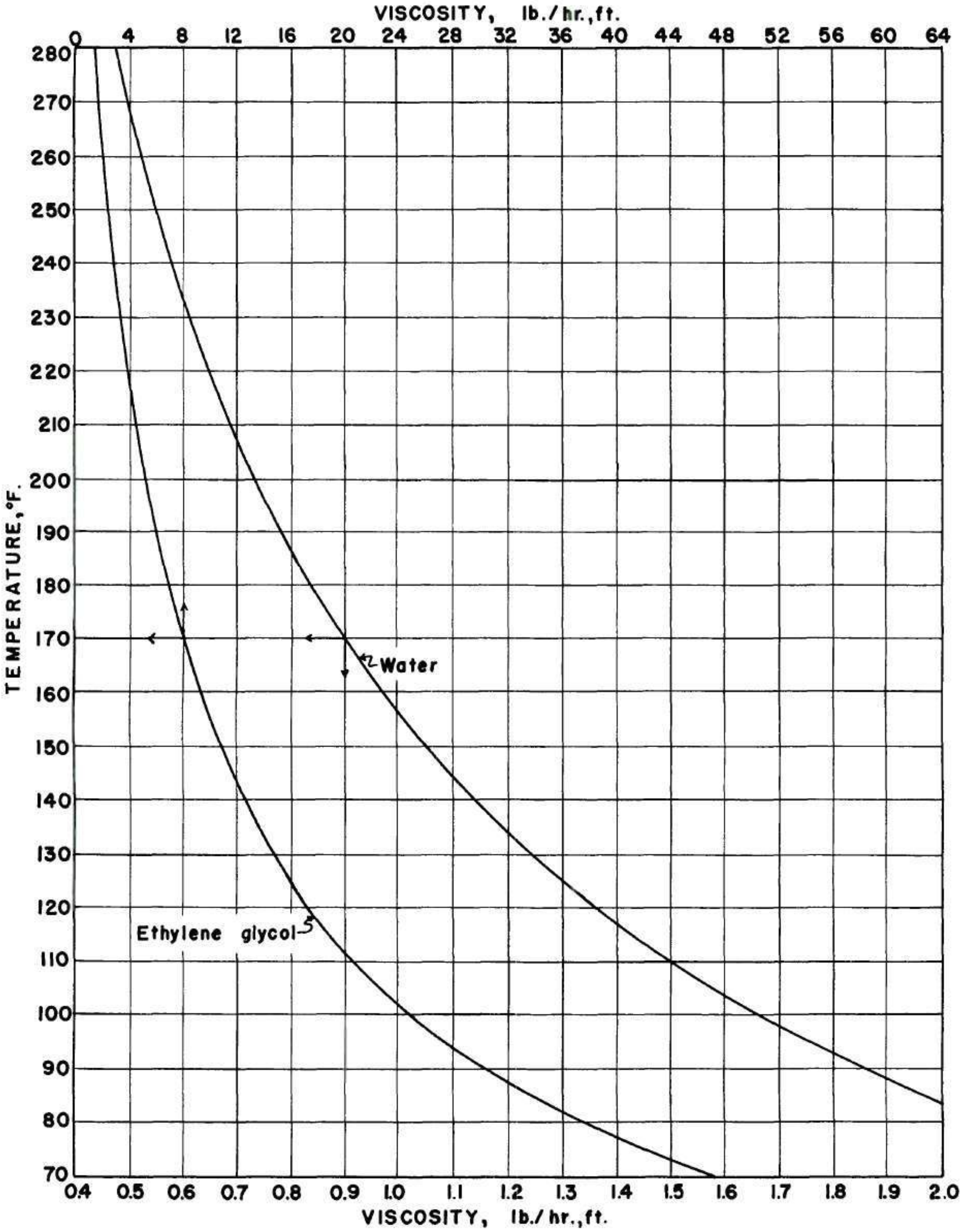


Figure 42. Viscosity of Water [Data from McAdams (1942)] and of Ethylene Glycol [Data from Carbide and Carbon Chemicals Corporation (1947)].

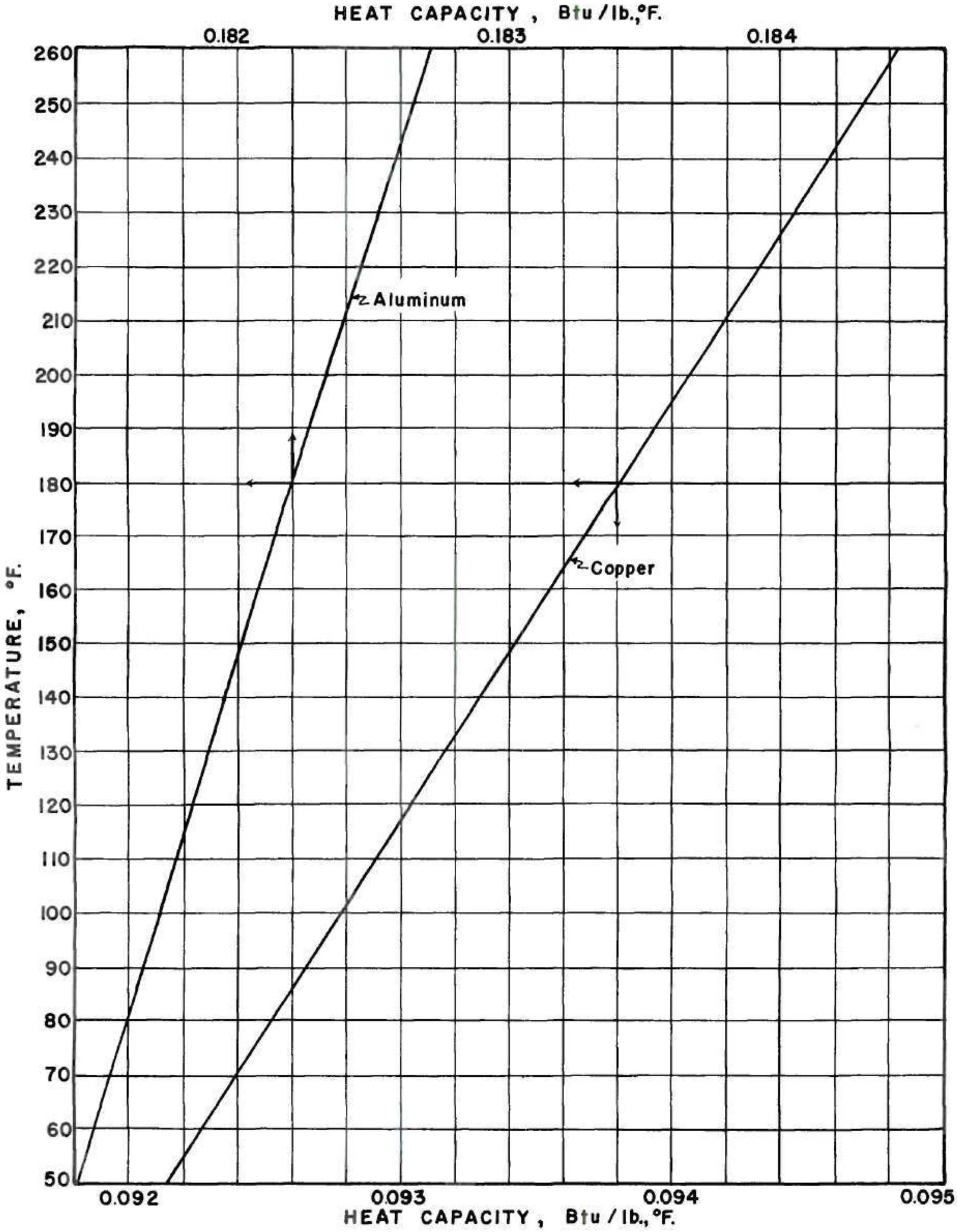


Figure 43. Heat Capacity of Copper and Aluminum [Data from McAdams (1942)].

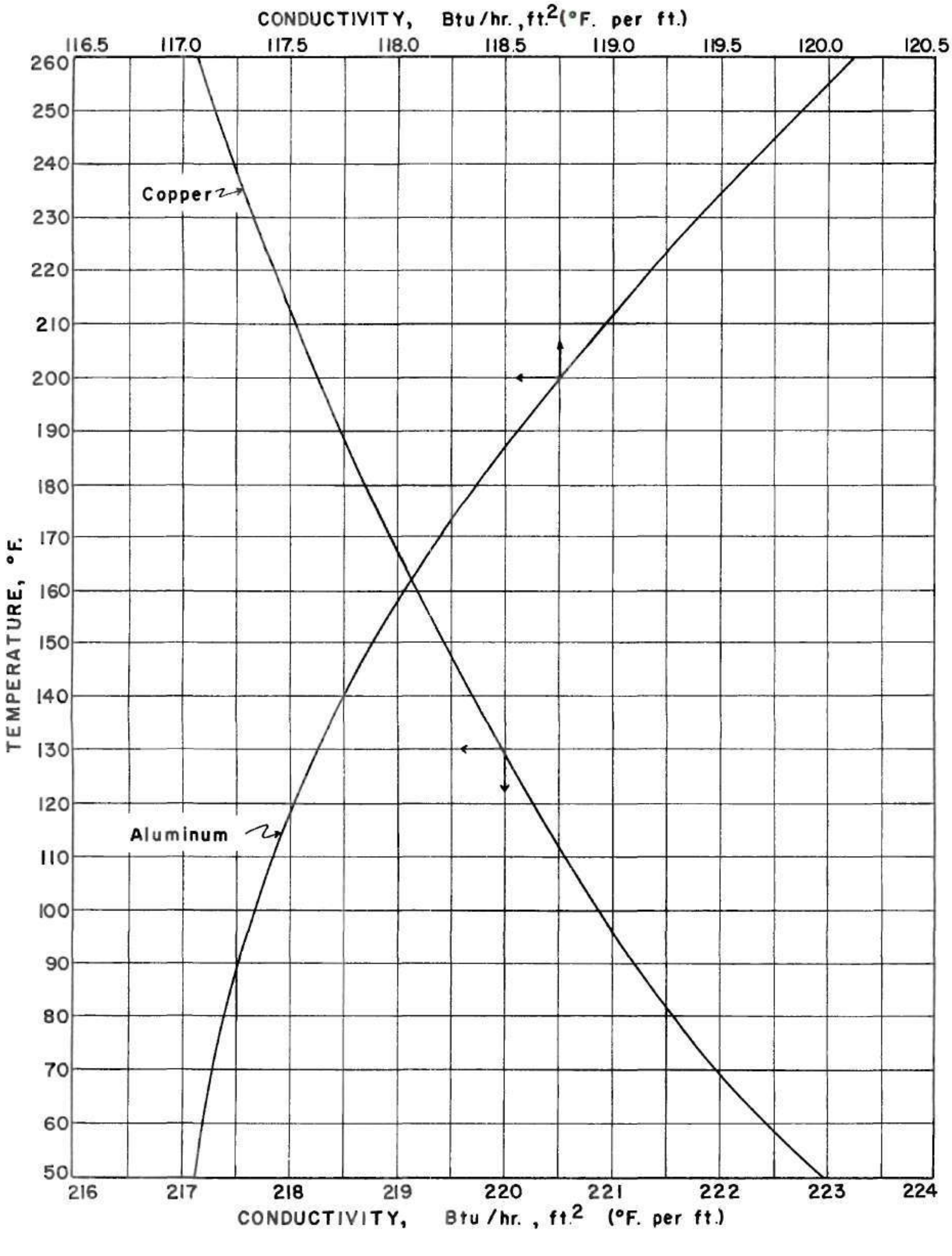


Figure 44. Thermal Conductivity of Copper and Aluminum [Data from McAdams (1942)]. .

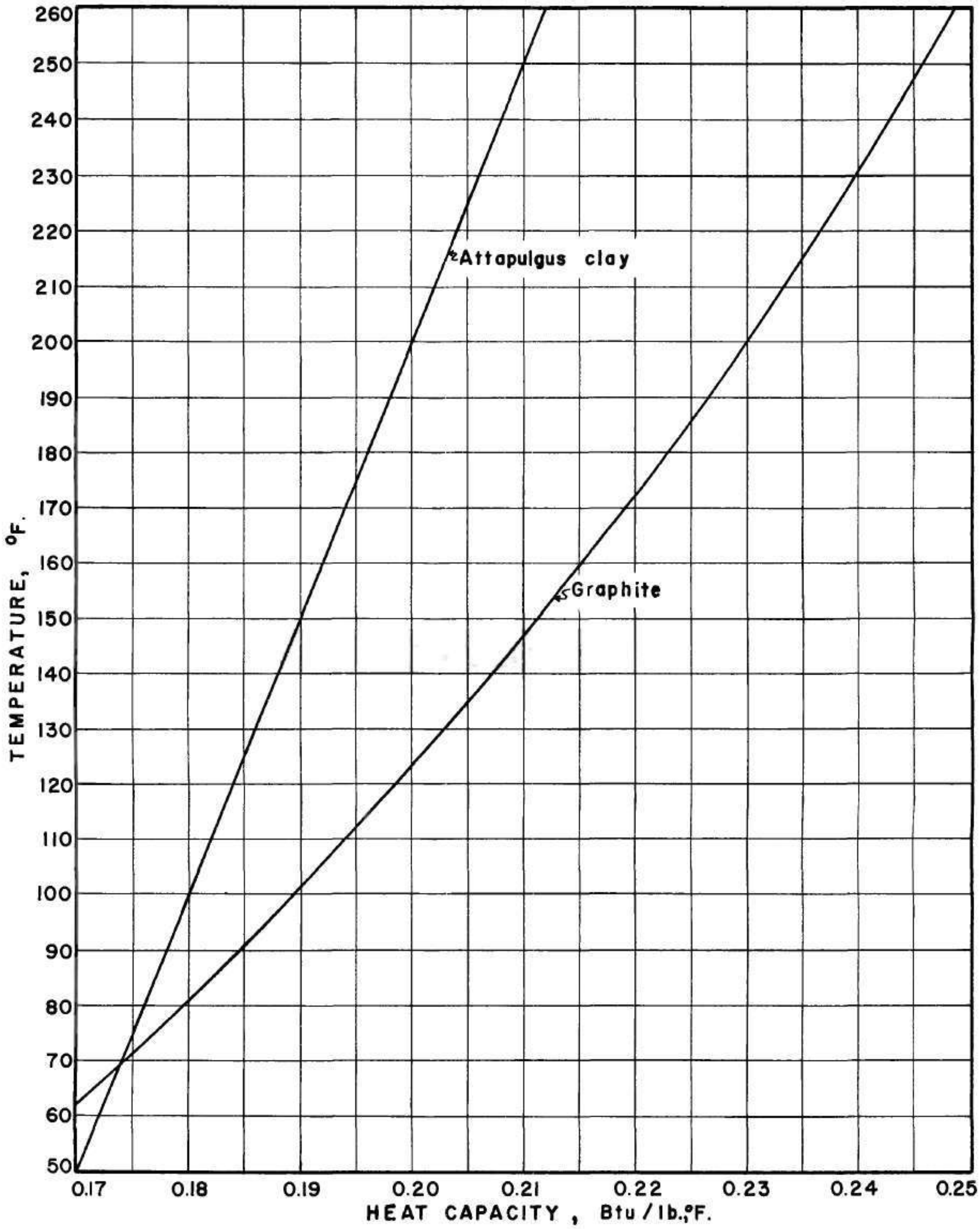


Figure 45. Heat Capacity of Graphite [Data from Perry (1950)] and of Attapulgus Clay [Data of Wert (1952)]..

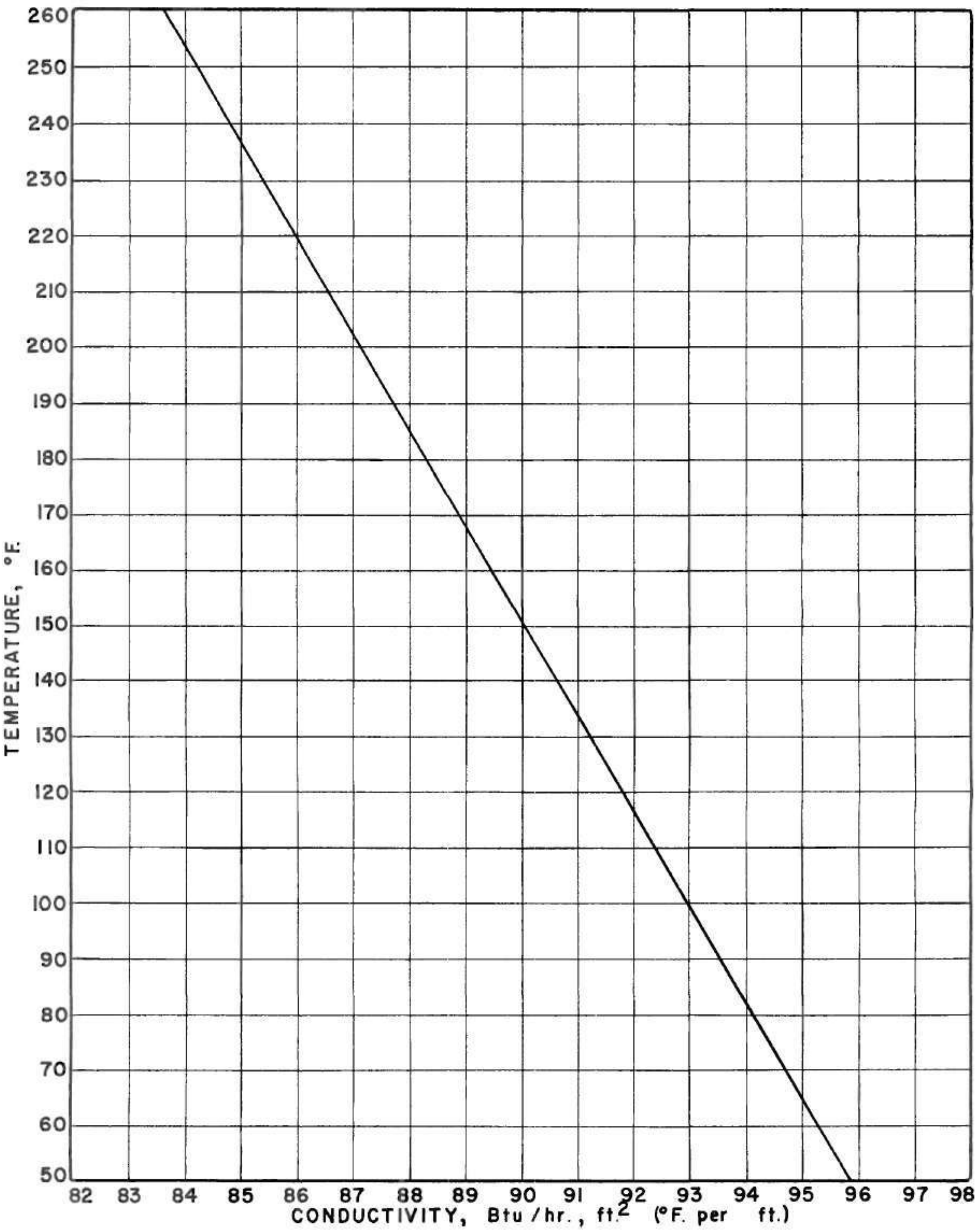


Figure 46. Thermal Conductivity of Acheson Graphite [(Data of Powell (1937)]. .

POISEUILLE'S LAW

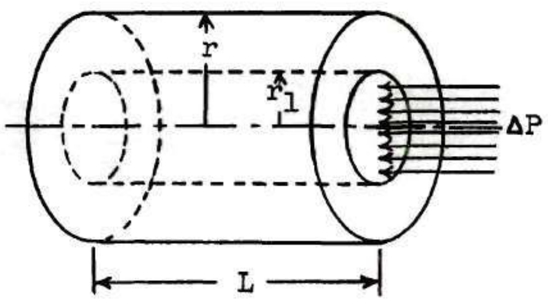
Sir Isaac Newton defined the coefficient of viscosity of an ideal liquid as the numerical value of the tangential force on a unit area of one of two parallel planes a unit distance apart when the space between the planes is filled with the liquid in question and one of the planes moves with unit velocity in its own plane relative to the other. Expressed mathematically, this is

$$F = \mu \frac{v}{x} , \tag{40}$$

where F is the force, v is the velocity, x is the distance of separation, and μ is called the coefficient of viscosity. Since the velocity of the liquid changes continuously, equation 40 may be written with differentials as

$$F = \mu \frac{dv}{dx} . \tag{41}$$

Consideration of equation 41 and of the conditions imposed when a liquid flows through a tube permits the derivation of a readily usable expression. With the tube horizontal so that the weight of the liquid is of no influence, let pressures be applied to both ends of the tube such that a net pressure, Δ P, is exerted as shown in the sketch. If the liquid



was originally at rest, it will move with ever-increasing velocity until the viscous resistance just balances the driving pressure. The condition for equilibrium may be written for the liquid cylinder of length L and radius r_1. The cross-sectional area of the cylinder is πr_1^2, and if ΔP is the pressure per unit area, the driving force is $\pi r_1^2 \Delta P$. The viscous resistance, F per unit area, acts on the sides of the cylinder of area $2\pi r_1 L$ with a total force of $2\pi r_1 LF$. Since equilibrium conditions prevail,

$$2\pi r_1 LF = \pi r_1^2 \Delta P$$

$$\text{or} \quad F = \frac{r_1 \Delta P}{2L} . \tag{42}$$

Therefore, using equation 41,

$$dv = \frac{\Delta P r_1 \, dr_1}{2L\mu} . \tag{43}$$

Equation 43 may be integrated, yielding

$$v = \frac{\Delta P r_1^2}{4L\mu} + C . \tag{44}$$

It has been found that, without exception, an ideal (or Newtonian) liquid adheres to the wall so that no slipping occurs; v is therefore zero when $r_1 = r$, and

$$v = \frac{\Delta P(r^2 - r_1^2)}{4L\mu} . \tag{45}$$

The velocity distribution, therefore, follows a parabola. As may be seen, dv/dr_1 vanishes for $r_1 = 0$, meaning that the parabola has a tangent normal to the axis there and that there is, furthermore, no cusp, corner, etc., at that point.

The maximum velocity at the center of the tube ($r_1 = 0$) is

$$v_{max} = \frac{\Delta P r^2}{4L\mu} . \qquad (46)$$

The volume of flow in a certain time is more convenient to use than the velocity of flow; therefore, by the well-known formula for the volume of a rotational paraboloid, the two quantities are related by

$$\frac{v}{t} = \frac{\pi r^2 v_{max}}{2} . \qquad (47)$$

Combining equations 46 and 47 results in the relationship which is known as Poiseuille's law and which, written in terms of a differential rate of flow and tube diameter, has been given the number 19 in the text. Thus,

$$v = \frac{\pi r^4 \Delta P t}{8L\mu} . \qquad (19)$$

Poiseuille's law was derived so that one important point might be brought out. As discussed previously in the text, equation 19 may be written in engineering units as

$$\mu = \frac{g_c D}{8v} \cdot \frac{D\Delta P}{4L} . \qquad (33)$$

The last term, $D\Delta P/4L$ or $r\Delta P/2L$, may be seen by equation 42 to be nothing but the shearing stress at the internal surface of the tube. The term $g_c D/8v$ is therefore the rate of shear at the same place.

TABLE X

THERMOCOUPLE CALIBRATION DATA*

Test Number	Temperature Indicated by Bureau of Standards Calibrated Thermometers (°F.)	Thermocouple Indication**					
		No. 1 (mv.)	No. 2 (mv.)	No. 3 (mv.)	No. 4 (mv.)	No. 5 (mv.)	No. 6 (mv.)
1	111.2	1.7714	1.7706	1.7730	1.7723	1.7739	1.7733
2	111.2	1.7757	1.7695	1.7740	1.7730	1.7692	1.7736
3	111.2	1.7745	1.7687	1.7737	1.7714	1.7663	1.7738
4	211.3	4.2371	4.2354	4.2354	4.2366	4.2365	4.2365
5	321.4	4.2159	7.2096	7.2145	7.2165	7.2145	7.2160
6	321.4	7.2200	7.2131	7.2020	7.2104	7.2227	7.2187

*Thermocouples Nos. 1 and 6 were fastened in copper tubes, and thermocouple No. 5 was embedded in the wall of a section of copper pipe.
**Reference junction, 0.0° F.

EXPERIMENTAL HEAT TRANSFER DATA

WATER AND #18 Glass Beads (Solid Material) DATE Nov. 22, 1951

RUN NO. 32

STEAM

Main line pressure, 151, 146, 146 psig; corrected ave. 161 psia.

Exchange-section pressure, 14.4, 14.5 psig; corrected ave. 28.8 psia.

Calorimeter temperature, 144.6, 144.7 °C.; average 292.4 °F.

Condensate rotameter reading, 100, 98, 97, 97 ; ave. flow rate 3.51 lb./min.

Outer jacket condensate flow rate, ________ lb./min.

Barometric pressure 29.40 in. of Hg.

SUSPENSION

Suspension rotameter reading, 201.5, 202.0 ; ave. flow rate ____ lb./min.

Suspension column pressure, 2.0 psig; corrected 16.4 psia.

Manometer readings, + 8.70 , - 9.30 in. Hg.; pres. drop 8.84 lb./in.^{2}

Suspension flow rate by direct weighing

lb.	oz.;	Tare, lb.	oz.;	Time, sec.
31	2	4	9	20
31	0	4	9.5	20
31	0	4	9.5	20

Average flow rate, 79.4 lb./min.

TEMPERATURES

Susp. inlet 1.	2.825 mv.	2.828 mv.	156.6 °F.	11.	4.795 mv.		231.9 °F.
2.	3.425		180.5	12.	4.798		232.0
3.	4.767		230.9	13.	4.795		231.9
4.	4.781		231.3	14.	4.796		231.9
5.	4.786		231.6	15.	4.796		231.9
6.	4.783		231.4	16.	4.792		231.8
7.	4.792		231.8	17.	4.807		232.3
8.	4.790		231.7	Susp. outlet 18.	4.100	4.098	206.2
9.	4.788		231.6	19.	2.861		158.1
10.	4.790		231.7	Condensate 20.	4.787		231.6
				Tank Thermometer.			155

Figure 47. Sample Data Record.

AUXILIARY EXPERIMENTAL DATA

WATER AND #18 Glass Beads (Solid Material) DATE Nov. 24, 1951

RUN NO. 32

DENSITY OF SOLID MATERIAL, 2.86 gm./cc., 178.5 lb./cu. ft.

SUSPENSION CONCENTRATION AND DENSITY

Weight of pycnometer and suspension	58.349	58.605	58.517	58.335	gm.
Weight of pycnometer	32.943	32.943	32.943	32.943	gm.
Weight of suspension	25.406	25.662	25.574	25.392	gm.
Temperature of suspension	27.6	27.4	27.0	27.2	°C.
Volume of suspension	22.763	22.763	22.763	22.763	cc.
Density of suspension	1.1161	1.1273	1.1235	1.1155	gm./cc.

Average density, 69.9 lb./cu. ft.

Weight per cent solids ______ ______ ______ ______

Average weight per cent solids 16.5

Volume per cent solids ______ ______ ______ ______

Average volume per cent solids 6.28 at 181.4°F

SUSPENSION VISCOSITY

Time of flow through viscometer, ______ ______ ______ ______ sec.

Temperature of suspension, ______ ______ ______ ______ °F.

Viscosity of suspension, __________ lb./hr. ft.

SUSPENSION THERMAL CONDUCTIVITY

T_1 ______ ______ ______

T_2 ______ ______ ______

T_3 ______ ______ ______

T_4 ______ ______ ______

Average thermal conductivity, ________ Btu./hr. ft. °F.

Figure 48. Sample Data Record.